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RELIABILITY GROWTH AND ITS
APPLICATIONS TO DORMANT
RELIABILITY

Thesis

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AFIT/GOR/MA/81D-12

RELIABILITY GROWTH AND ITS APPLICATION TO DORMANT RELIABILITY

THESIS

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Preface

This thesis is an initial attempt to use reliability growth models to predict dormant reliability. It can actually be thought of as a two-part thesis. The first part pertains to reliability growth, and the second part to dormancy and the Monte Carlo experiments.

The purpose of this study is to provide evidence that it is possible to predict dormant reliability with reliability growth models. My hope is that the initial work done in this thesis can someday be expanded to provide a sound methodology for analyzing dormant reliability.

The original idea of using reliability growth models to predict dormant reliability was conceived by my Thesis Advisor, Professor Albert H. Moore. I am sincerely indebted to him for his inspiration and help in completing this study. Also, I would like to thank the Reader of my thesis, Lt. Col. Edward J. Dunne, whose constructive criticism was an important part of the final report. Finally, I would like to express great thanks to my wife, Jackie, whose hours at the typewriter played an essential role in the completion of this project.

John F. VonLoh

Contents

Page

List of Figures

List of Tables

Table		Page

AFIT/GOR/MA/81D-12

Abstract

This thesis presents the results of an extensive literature search into reliability growth and the subsequent use of three reliability growth models to predict dormant reliability. A brief review of reliability theory is followed by a survey of reliability growth models, which includes the detailed developments of and specific examples for five popular models. The nature of dormant reliability is then discussed as a prelude to a Monte Carlo analysis using the Duane, Gompertz, and Bonis reliability growth models to predict dormant reliability.

RELIABILITY GROWTH AND ITS APPLICATIONS

TO DORMANT RELIABILITY

I. Introduction

Reliability of dormant weapon systems has been a concern throughout military history (Ref 32:7). Many strategic weapon systems in the Air Force today, for example, lie in a dormant state for years, but are expected to perform reliably when needed. Many of these systems are vitally important to our national security. In addition, there is always a cost associated with the replacement or repair of systems when they fall below acceptable reliability levels. Since reliability has a direct impact on the mission performance and operating cost of a system, it follows that there is a need to assess the impact of dormancy on a weapon system.

One of the problems with dormancy is the lack of consistent or well-defined methods for determining its effects (Ref 32:11). This fact is reinforced by remarks in the Introduction of a report by J. Bauer, D.F. Cottrell, T.R. Cagnier, and E.W. Kimball, of the Martin-Marietta Corporation. They stated:

> ...Documents such as RADC Reliability Notebook and MIL-HDBK-217A depict in detail operational failure rate data, derating factors, environmental factors, quality factors, etc. Little or nothing is extant

on the other states of activation--storage, dormancy, and power on-off cycling [Ref 8:1-1].

These statements are evidence that there is knowledge to be gained by additional research into dormant reliability.

A general definition of dormant reliability is the change in reliability of a system over time as it lies in a dormant or unused state (Ref 32:22). There is no single or consistent definition of exactly what constitutes a dormant state. However, most of the literature will refer to a dormant state as a state of very little or no operational stress (Ref 24:45). For example, a missile system to which enough power is added to see if the components are functioning properly, say once a month, could be considered a dormant system. Of course, the effect of dormancy on a system will vary with the system and the environment in which it is stored. In most cases, however, the effect of dormancy is an increase in the failure rate of the system with the passage of time.

On the other hand, a general definition of reliability growth is a continuing decrease in a system's actual failure rate that will approach an inherent value of the system (Ref 31:330-331). The inherent value of the system refers to the maximum reliability one can expect from a system based on the design characteristics. A good example of reliability growth would be getting the "bugs" out of a new car. Once the early problem areas are fixed, the car will perform with a higher level of reliability for most of its life.

Compared with reliability growth, dormant reliability has an opposite or negative effect. With reliability growth the failure rate will be decreasing, while with dormant reliability the failure rate will be increasing. Intuitively then, it seems that the methodology used to determine reliability growth could be applied to dormant reliability.

In general, the nature of reliability growth modeling is to first test a system, then analyze the results of the tests, and finally fix the system (Ref 14:35). The procedure is repeated until enough data are obtained to construct a reliability growth curve. From the reliability growth curve, estimations of future reliability of the system can be made. Analogous to this situation is a dormant system which is stored and tested at regular intervals. In both the dormant case and the case where reliability growth is present, the objective is to determine the change in the failure rate of the system. This analogy should make it possible to use reliability growth models to estimate dormant reliability.

Since the theme of this study hinges on an in-depth understanding of reliability growth, the first objective will be to present a survey of applicable reliability growth models. Included in the survey will be a detailed discussion of five reliability growth models which represent the most widely used models that are capable of predicting future reliability. In addition, there will be a general discussion of other reliability growth models that were found in the literature search.

In light of the analogy between reliability growth and dormant reliability (each has changing failure rates), research to determine whether reliability growth models can be used to estimate dormant reliability would be important. This is because much work has already been done in reliability growth. This reasoning supports the second objective of this study, that is to determine with relative "goodness-of-fit" measures how well a set of selected reliability growth models fit dormant data generated by Monte Carlo simulation. Simulation was used so the underlying failure distributions would be known and could be used to calculate the true system reliability. The true system reliability was used as the basis for comparison in the relative "goodness-of-fit" measures.

To achieve the study objectives, an intensive literature search was made into reliability growth and dormant reliability. The results of this literature search are presented in the following sections: Section II summarizes some of the key relationships associated with reliability theory which are used throughout the study; Section III discusses reliability growth models; Section IV is a general discussion of the nature of dormant reliability; Section V outlines the Monte Carlo simulation; Section VI presents the results of the Monte Carlo experiments; and finally, Section VII contains conclusions to the study and recommendations for further research. In addition, the Appendix contains additional significant data resulting from the Monte Carlo simulations

II. Reliability Review

This section is a short review of some of the reliability concepts that will be referred to in this study. It is intended to refamiliarize the reader with the important probabilistic relationships used in reliability theory. In addition, emphasis will be placed on general notation that is used in this report.

The Reliability Function

The probability of a system failure as a function of time (t) is defined as

$$P(T \leq t) = F(t), \quad t \geq 0 \tag{2.1}$$

where T is a random variable denoting time to failure. F(t) is the cumulative density function (cdf) of the failure times, or the probability that the system will fail by time t.

The reliability of the system at time t is defined as

$$R(t) = 1 - F(t) = P(T > t) \tag{2.2}$$

where R(t) is the reliability function. If the random variable T has a probability density function (pdf) equal to f(t), then

$$R(t) = 1 - F(t) = 1 - \int_0^t f(x)\,dx = \int_t^\infty f(x)\,dx \tag{2.3}$$

where x is used as the variable of failure time before integration takes place. Eq (2.3) is the fundamental relationship between the pdf - $f(t)$, the cdf - $F(t)$, and the reliability function - $R(t)$ (Ref 30:9-10).

In this study, the reliability will usually be denoted by the letter R. For example, the reliability as a function of time would be $R(t)$, and the reliability at a specific time, say $t = 15$, would be $R(15)$. This notation is consistent with most textbooks and articles on the subject.

The Expected Life

The expected life of a system is the time during which it is expected to perform successfully (Ref 30:10). If the random variable T is used to denote time to failure, then by definition

$$E(T) = \int_0^{\infty} t f(t)dt \qquad (2.4)$$

where $E(T)$ is the expected value of the random variable T (Ref 38:121). Another convenient expression for the expected life is

$$E(T) = \int_0^{\infty} R(t)dt \qquad (2.5)$$

This expression is derived as follows. Let $u = R(t)$ and $dv = dt$. Then

$$du = d[R(t)] = d[1-F(t)] = d[-F(t)] = -f(t)dt$$

and $v = t$. Using integration by parts,

$$\int_0^\infty R(t)dt = \int_0^\infty u dv$$

$$= uv - \int_0^\infty v du$$

$$= [tR(t)] \Big|_0^\infty + \int_0^\infty t f(t) dt \qquad (2.6)$$

it is clear that when t=0, the first term of (2.6) is also zero. In addition, it is assumed that R(t) = 0 at t = + ∞ (reliability is a decreasing function of time) and the first term is again zero. This leaves only the second term

$$\int_0^\infty t f(t) dt = E(T) \qquad (2.7)$$

from Eq (2.4).

Terms associated with E(T) are the mean time to failure (MTTF) or mean time between failure (MTBF). However, these terms should be used only when the failure distribution function is specified. The reason for this is because the reliability at the MTBF or MTTF is not generally the same for any two given distributions. For example, the R(MTBF) for normal density function is $P(z \geq 0) = .5$. However, R(MTBF) for the negative exponential model is EXP(-1) = .368 (Ref 30:11).

Relationship to the Hazard Function

The hazard function, h(t), is defined as the instantaneous failure rate of a system at time t. More formally, it is the conditional probability that a system fails in the small interval $(t, t+\Delta t)$, given it has survived until time t. The relationship between h(t), f(t), F(t), and R(t) is as

follows

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{f(t)}{R(t)} \tag{2.8}$$

The derivation of this relationship can be found in Ref 30: 11-15 or Ref 31:135-137.

A special case, when the hazard function is constant throughout time, is the case of the neg exponential distribution having the form (Ref 30:234)

$$R(t) = EXP(-\lambda t), \quad t \geq 0 \tag{2.9}$$

where

R(t) = the reliability at time t

λ = the instantaneous failure rate

The parameter of the model, usually denoted by λ, is the hazard rate for all t (Ref 30:234). This has led engineers to refer to the failure rate as λ or $\lambda(t)$ in many cases. However, care must be taken to assure that the failure rate referred to is actually the instantaneous failure rate and not a cumulative failure rate or failure rate over a specified interval. It must be emphasized that the relationship in Eq (2.8) only applies when the instantaneous failure rate is used. In this study, the instantaneous failure rate will be denoted by h(t) when the relationship in Eq (2.8) is expected to hold.

III. Reliability Growth

The failure rate of a system in its early life or development stage is often characterized by change (Ref 30: 3). Many factors can cause this change. For example, an equipment may have a faulty component that is redesigned or replaced with a better component which will cause the equipment to be more reliable after the change. Another example would be the increase in reliability of a weapon system, like a new aircraft, that would occur after the operators became more familiar with how it worked. This phenomenon is called "reliability growth."

As the name implies, the general notion is that the reliability will "grow" to some inherent value (Ref 31:330-331). The inherent value could either be a reliability target set for the system, or it could be a limiting reliability based on the design of the system. However, an increase in reliability need not be the only case. A change could be made that would be detrimental to the system. The point is that a system in its early life is generally not characterized by random failures of a constant nature, but rather by a changing failure rate that hopefully will increase with time.

The Survey

An intensive investigation into reliability growth

was conducted for two reasons. The first reason was to determine if reliability growth modeling techniques had ever been used in the past to predict dormant reliability. Secondly, an in-depth understanding of reliability growth was needed for the experiments on dormant reliability in the latter part of this study.

The literature search revealed numerous reliability growth models. Some models can be used to predict future reliability levels, while others cannot. In addition, some models are quite easy to use, while others require the use of more complicated procedures. For example, the Duane model (Ref 22) uses a simple graphical technique to estimate future reliability, while Singpurwalla's technique (Ref 53) requires use of the Box and Jenkins time series analysis procedures (Ref 11), which are more complicated. For these reasons, not every model was studied in great detail.

To give the reader a good idea of what reliability growth involves, five models are discussed in detail in this survey. These models are: 1) the Duane model; 2) the Crow model; 3) Lloyd and Lipow's hyperbolic model; 4) the Gompertz model; and 5) the Bonis model for one shot devices. These models were chosen because they are widely used by military analysts, and because they provide a good cross-section of reliability growth modeling techniques. Also, they were chosen because they each provide a predictive functional relationship for the reliability growth curves. This feature makes it possible to predict reliability levels of

future times t. Some models, such as the Barlow and Scheuer model (Ref 5), do not have this capability.

There will also be a section of this chapter devoted to other reliability growth models. Included in this section will be a brief description of many of the other reliability growth models. The intent is to inform the reader, in a very general way, about each of these models and to provide references which discuss each model in detail.

The Duane Model

J.T. Duane, of the General Electric Company, developed a reliability growth model that has been widely used in development and testing programs (Ref 22:563-566). The model has been used extensively by the military since its first publication in 1962. The idea for the model came when Duane noticed that the plot of cumulative mean times between failure (MTBF) for successive test intervals would many times plot as a straight line on log-log paper (Ref 17:6). Duane used the plots to determine the change in failure rates as a function of time.

The Model. The basic assumption of the Duane model is that a testing program is divided into intervals (Ref 19: A1-A6). Each interval could represent a design change or engineering modification, but the failure rate for any given interval would remain constant during that interval. This assumption implies that the reliability during any specific interval can be represented by the negative exponential model,

$$R_i(t) = \mathrm{EXP}[-\lambda_i t], \quad t \geq 0 \tag{3.1}$$

where

$R_i(t)$ = the reliability for the i^{th} interval

λ_i = the failure rate for the i^{th} interval

t = the time

i = 1,2,..., the total number of intervals

Mathematically, the form of the Duane model is

$$\lambda_c(t) = \frac{N(t)}{t} = Kt^{-\alpha}, \quad t \geq 0,\ K \geq 0 \tag{3.2}$$

where

$\lambda_c(t)$ = the cumulative failure rate

$N(t)$ = the cumulative number of failures

t = the cumulative test time

α = the growth rate constant

K = constant that represents the cumulative failure rate at $t = 0$

The instantaneous failure rate can be derived by taking $dN(t)/dt$. This yields

$$N(t) = Kt^{1-\alpha}$$

$$\lambda(t) = \frac{dN(t)}{dt} = (1-\alpha)Kt^{-\alpha} \tag{3.3}$$

where $\lambda(t)$ is the instantaneous failure rate. From this equation the reliability can be calculated for a time t that corresponds to the i^{th} interval and, thus, $\lambda(t)$ becomes λ_i in Eq (3.1).

Parameter Estimation. Taking logarithms of Eq (3.2) gives the mathematical representation of the straight-line

plots on log-log paper (Ref 19:A4). The resulting equation is

$$\ln\left[\frac{N(t)}{t}\right] = \ln(K) - \alpha\ln(t) \qquad (3.4)$$

From this equation, two methods of parameter estimation emerge (Ref 4:12-14). First, least squares can be used by taking the ln(t) and the ln[N(t)/t] as input. Second, a graphical technique can be employed where K is estimated as the intercept of the plot at $t = 1$ or $\ln(t) = 0$, and α is estimated as the negative of the slope of the line.

Least squares may be the more exact method of the two (Ref 4:13), but the graphical technique lets the analyst visualize the process (Ref 15:458-459). By getting a visual perspective, some additional insight may be gained. For example, least squares techniques would not point out a specific outlier in the data; however, if the graphical technique was used, the outlying data point could be easily seen.

Example. Suppose the data listed in Table I contain the cumulated time (t) and cumulated failures [N(t)] of a test program (Ref 19:A5). The cumulated time is the total test time for the item or items on test. For example, if three items are placed on test for one hour, the cumulated time is three hours. In this case, three items were placed on test, and as a failure occurred, appropriate design changes were made to all three items. They were then put back on test until $t = 10$. Using log-log graph paper, t is plotted on the abscissa and N(t)/t on the ordinate. The

TABLE I

Cumulated Test Data

t	N(t)	N(t)/t
1.00	3	3.00
2.00	6	3.00
5.00	13	2.60
8.00	18	2.25
10.00	22	2.20

slope is then computed and used as the estimate for α. In this case, the slope is .148 = 14.0mm/94.3mm. The estimate for K is simply the intercept (ordinate value) at t = 1, which is 3.18. Figure 1 shows the plot of the data as well as the numbers used to arrive at the estimates for α and K.

If least squares are used to estimate the parameters, ln(t) is considered the independent variable and ln[N(t)/t] the dependent variable (from Eq 3.4). The results of using least squares estimators on the data in Table I are α = .147 and K = 3.15. These agree closely with the values determined using graphical techniques.

Once values for K and α have been found, instantaneous failure rates can be calculated using Eq (3.3). Applying the negative exponential model, Eq (3.1), will provide estimates of reliability at any specified time. However, one must realize that future predictions are based on the rate of growth or deterioration which was determined from the

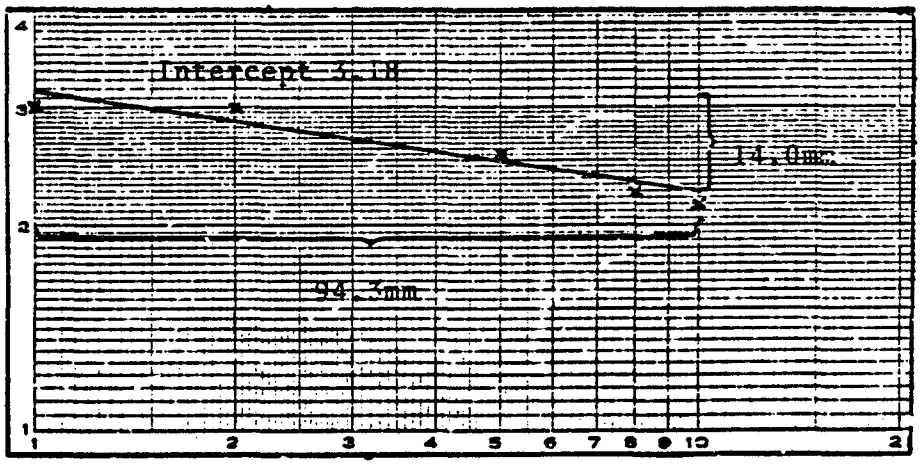


Fig 1. Plot of Data on Log-Log Paper

original data. In other words, the failure rate will change consistently with time.

The least squares method of estimation assumes both independence of the failure times and also constant variance (Ref 37:389). Since this is usually not the case with cumulative data, confidence limits using least squares properties are not statistically valid (Ref 12:3). However, the Crow model, which will be discussed next, uses maximum likelihood estimators which do enable the analyst to calculate confidence limits.

The Crow Model

Larry H. Crow, while working with the U.S. Army Materiel Systems Analysis Agency (AMSAA), took the Duane model a step further. Crow used the mathematical interpretation of the straight-line plots of the cumulated failure rate on log-log paper to show that system failure times were following a nonhomogeneous Poisson process with Weibull

intensity (Ref 16:205-212).

The Model. The straight-line plot of cumulated failure rates versus cumulated time on log-log paper means

$$\ln\left[\frac{N(t)}{t}\right] = a + b\ln(t) \tag{3.5}$$

which is Eq (3.4) of the Duane model where ln(K) = a and $-\alpha = b$ (Ref 19:A2-A3). Equating N(t) with its expected value (assuming an exact linear relationship) and taking exponentials gives

$$\frac{E[N(t)]}{t} = e^{a}t^{b} \tag{3.6}$$

or

$$E[N(t)] = e^{a}t^{b+1} \tag{3.7}$$

Letting $B = b + 1$ and $\alpha = e^{a}$ yields

$$E[N(t)] = \alpha t^{B} \tag{3.8}$$

The instantaneous failure rate, h(t), is obtained by differentiating Eq (3.8) with respect to t. This gives

$$h(t) = dN(t)/dt = \alpha B t^{B-1} \tag{3.9}$$

which is recognized as the Weibull hazard function. Of course, the assumption that failure rates remain constant for the duration of any interval is still in force. This implies that failures follow a nonhomogeneous Poisson process with Weibull intensity function h(t). In other words, for the duration of any interval i, Eq (3.1) applies. This is consistent with the assumption of the Duane model,

however, the changing parameter, λ_i, is modeled with the Weibull hazard function (Eq 3.9).

Parameter Estimation. The main difference between the Crow model and the Duane model lies in the method of parameter estimation (Ref 16:207). When using least squares, it is assumed that the data are independent with constant variance. Crow points out that the Duane model assumes the cumulated failure rates are consistently increasing, decreasing, or remaining the same. This implies the data are not independent (Ref 16:206). In addition, the variance of the cumulative failure rate is not constant, but decreases as time increases (Ref 16:206). The result is that confidence bounds of α and B using least squares properties are not valid.

Rather than using least squares or linear estimations from log-log paper, Crow uses maximum likelihood (ML) estimators. The form of the ML estimators are

$$\hat{B} = \frac{N}{\sum_{i=1}^{N-1} \ln\left[\frac{t_n}{t_i}\right]} \tag{3.10}$$

and

$$\hat{a} = \frac{N}{t_n^{\hat{B}}} \tag{3.11}$$

where

N = the total number of failures observed

t_i = time at the i^{th} failure

t_n = time at the N^{th} failure

These estimators assume a single reparable system with failures that follow a nonhomogeneous Poisson process with Weibull intensity. When a failure occurs, the system is repaired and put back into service, and the repair time is considered negligible (Ref 18:383-388).

The existence of the ML estimators make it possible to calculate confidence intervals (Ref 16:206). In addition, Crow suggests using the Cramer-von Mises statistic as a test for the appropriateness of the model.

Example. Table II represents data from a test program truncated after the N^{th} failure (Ref 19:A11). The first step is to estimate B and α using the ML estimates, and then perform a goodness-of-fit test using the Cramer-von Mises test statistic (Ref 19:A12). The ML estimate of B is

$$\hat{B} = \frac{N}{(N-1)\ln(t_n) - \sum_{i=1}^{N-1} \ln(t_i)} = \frac{15}{14(6.856)-63.456} = .4611 \quad (3.12)$$

The ML estimate for α is

$$\hat{\alpha} = \frac{N}{t_n^{\hat{B}}} = \frac{15}{949.7^{.4611}} = .6355 \quad (3.13)$$

The Cramer-von Mises statistic is (Ref 19:A10)

$$C_M^2 = \frac{1}{12M} + \sum_{i=1}^{M}\left[\left(\frac{t_i}{t_n}\right)^{\bar{B}} - \frac{2i-1}{2M}\right]^2 \quad (3.14)$$

where

M = N-2, where N is the number of failures

TABLE II

Failure Data for Crow Model

Failure Number	Failure Time	$\ln(t_i)$	$\Sigma\ln(t_i)$
1	1.5	.405	.405
2	3.2	1.163	1.569
3	11.8	2.468	4.037
4	29.6	3.388	7.424
5	53.6	3.982	11.406
6	65.2	4.177	15.583
7	119.4	4.782	20.366
8	265.3	5.581	25.947
9	294.0	5.684	31.630
10	441.1	6.089	37.720
11	465.1	6.142	43.862
12	567.0	6.340	50.202
13	685.8	6.530	56.733
14	831.4	6.723	63.456
15	949.7	6.856	------

t_i = time of i^{th} failure

t_n = time of the N^{th} failure

$\bar{B} = \frac{M-1}{M}\hat{B}$, the unbiased estimate of B

C_M^2 = .0193 compared with a critical value of .169, therefore the model cannot be rejected. It is also possible to use the chi-square goodness-of-fit test for this model (Ref 13:49).

Since the model is appropriate, the instantaneous failure rate for the system may be estimated by

$$\hat{h}(t) = \hat{\alpha}\hat{B}t^{\hat{B}-1} \tag{3.15}$$

or

$$\hat{h}(t) = .6355(.4611)t^{-.5389} \tag{3.16}$$

This equation will produce point estimates for given values of t. In addition, confidence intervals can be computed for $\hat{\alpha}$, $\hat{B}$, and $\hat{h}(t)$ (Ref 19:A12-A13).

This model has found wide acceptance, especially by the military and military contractors. The Army Materiel Development and Readiness Command (DARCOM) pamphlet P702-4 has several numerical examples of applications of the Crow model (sometimes referred to as the AMSAA model) (Ref 19). In addition, Donald P. Amiotte did a study of how well this model tracks data (Ref 2).

The Lloyd and Lipow Hyperbolic Model

David K. Lloyd and Miron Lipow considered a reliability growth model based on the following assumptions (Ref 31:338-347). First, a test program is conducted in N stages, each stage consists of a certain number of tests or trials of the system in question, and the only data recorded is whether the system was a success or failure. In addition, the system has a fixed reliability during any particular stage and the results of the testing in a stage are used to improve the system in the subsequent stage. After the N^{th}

stage, a reliability growth curve is fit to the data.

The Model. The form of the growth function at the k^{th} stage is

$$R_k = R_\infty - \frac{\alpha}{k} \tag{3.17}$$

where

R_k = the reliability after the k^{th} stage of testing

R_∞ = the ultimate value of reliability which could be attained as k approaches infinity

α = parameter which modifies the growth rate

Parameter Estimation. There are two methods of parameter estimation suggested by Lloyd and Lipow (Ref 31: 337-347). The preferred method is maximum likelihood (ML), but the ML estimators involve two equations which must be solved iteratively by trial and error. This process could take a great deal of time without good initial values. This leads to the second method of estimation, least squares. Once the least squares estimators are found, they can be used as initial values for the ML estimators.

Since the data are comprised of S_k successes at the k^{th} stage, the likelihood at the k^{th} stage is given by

$$L_k = CR_k^{S_k} (1-R_k)^{N_k-S_k} \tag{3.18}$$

where

L_k = the likelihood at the k^{th} stage

C = a constant

$R_k = S_k/N_k$ = reliability at the k^{th} stage

S_k = the number of successes at the k^{th} stage

N_k = the number of items tested at the k^{th} stage

Assuming each stage is independent gives the likelihood function

$$L = \prod_{k=1}^{N} L_k = C' \prod_{k=1}^{N} R_k^{S_k}(1-R_k)^{N_k-S_k} \qquad (3.19)$$

The log-likelihood function is

$$\ln(L) = \ln(C') + \sum_{k=1}^{N} S_k \ln(R_k) + \sum_{k=1}^{N} (N_k-S_k)(1-R_k) \qquad (3.20)$$

Substituting $R_\infty - \frac{\alpha}{k}$ for R_k from Eq (3.17) yields

$$\ln(L) = \ln(C') + \sum_{k=1}^{N} S_k \ln(R_\infty - \tfrac{\alpha}{k})$$
$$+ \sum_{k=1}^{N} (N_k-S_k)\ln(1-R_\infty+\tfrac{\alpha}{k}) \qquad (3.21)$$

Taking partial derivatives with respect to each parameter gives the likelihood equations

$$\frac{\partial \ln(L)}{\partial R_\infty} = \sum_{k=1}^{N} \frac{S_1}{R_\infty - \frac{\alpha}{k}} - \sum_{k=1}^{N} \frac{N_k - S_k}{1 - R_\infty + \frac{\alpha}{k}} = 0 \qquad (3.22)$$

and

$$\frac{\partial \ln(L)}{\partial \alpha} = -\sum_{k=1}^{N} \frac{S_k/k}{R_\infty - \frac{\alpha}{k}} + \sum_{k=1}^{N} \frac{(N_k-S_k)/k}{1 - R_\infty + \frac{\alpha}{k}} = 0 \qquad (3.23)$$

These equations can be solved by trial and error to obtain the ML estimates for R_∞ and α.

As mentioned earlier, initial values for α and R_∞ can be determined using least squares techniques. To

determine the least squares estimators, let Q be the sum of the squared deviations of the observed success-ratio (S_k/N_k) from its expected value $(R_\infty - \alpha/k)$. This gives

$$Q = \sum_{k=1}^{N} (S_k/N_k - R_\infty + \alpha/k)^2 \qquad (3.24)$$

Taking partial derivatives with respect to each parameter gives the following two linear equations in two unknowns,

$$\sum_{k=1}^{N} \frac{S_k}{N_k} = NR_\infty - \alpha C_1 \qquad (3.25)$$

and

$$\sum_{k=1}^{N} \frac{S_k}{kN_k} = R_\infty C_1 - \alpha C_2 \qquad (3.26)$$

where

$$C_1 = \sum_{k=1}^{N} \frac{1}{k} \qquad (3.27)$$

$$C_2 = \sum_{k=1}^{N} \frac{1}{k^2} \qquad (3.28)$$

Solving Eqs (3.25) and (3.26) for α and R_∞ gives

$$\hat{R}_\infty = \frac{C_2 \sum_{k=1}^{N} (S_k/N_k) - C_1 \sum_{k=1}^{N} (S_k/kN_k)}{NC_2 - C_1^2} \qquad (3.29)$$

and

$$\hat{\alpha} = \frac{C_1 \sum_{k=1}^{N} (S_k/N_k) - N \sum_{k=1}^{N} (S_k/kN_k)}{NC_2 - C_1^2} \qquad (3.30)$$

where $\hat{\alpha}$ and $\hat{R}_\infty$ replace α and R_∞ respectively.

Example. Suppose 10 items were tested each week for 10 consecutive weeks. After each test, design changes were made to improve the system based on the results of the test. Table III is a summary of the data collected in this development program. When N (number stages) is 10, $C_1 = \Sigma\frac{1}{k} = 2.93$ and $C_2 = \Sigma\frac{1}{k^2} = 1.55$. The least squares estimates are

$$\hat{R}_\infty = \frac{1.55(7.2) - 2.93(1.59)}{10(1.55) - (2.93)^2} = .938 \qquad (3.31)$$

$$\hat{\alpha} = \frac{2.93(7.2) - 10(1.59)}{10(1.55) - (2.93)^2} = .745 \qquad (3.32)$$

The resulting growth function is

$$\hat{R}_k = .938 - (.745/k) \qquad (3.33)$$

This equation will give the reliability for any of the N stages by substituting in the appropriate value for k.

Of course, Eq (3.33) was found using least squares. If the ML estimators are desired, the values calculated for $\hat{\alpha}$ and $\hat{R}_\infty$ should be used as initial values in Eqs (3.22) and (3.23). Values for $\hat{\alpha}$ and $\hat{R}_\infty$ can be changed slightly with each iteration until Eqs (3.22) and (3.23) are both as close to zero as possible. The resulting values of $\hat{\alpha}$ and $\hat{R}_\infty$ are the ML estimators. With the properties associated with ML estimates, the analyst can also find confidence limits for the estimators and the reliabilities.

Lloyd and Lipow also briefly discuss two variations to the growth model shown in Eq (3.17). One can be used

TABLE III

Data from Test Program

k	N_k	S_k	kS_k	S_k/N_k	S_k/kN_k
1	10	3	3	.3	.300
2	10	4	8	.4	.200
3	10	7	21	.7	.230
4	10	6	24	.6	.150
5	10	7	35	.7	.140
6	10	8	48	.8	.130
7	10	8	56	.8	.114
8	10	9	72	.9	.113
9	10	10	90	1.0	.111
10	10	10	100	1.0	.100

when data are given as time-to-failure rather than merely the number of successes (Ref 31:347). The other uses binomial data, but weighs the current estimate of the reliability on the k^{th} trial with the estimates of all previous trials (Ref 31:348).

Lloyd and Lipow also developed a two-state model (perfect or imperfect) (Ref 31:331-338). However, this model is restricted by the assumption that failures can occur in only one way. In other words, only one failure mechanism is allowed when using the model. In addition, no consistent estimators for the parameters are provided (Lloyd and Lipow do not provide any, and Sherman, who

conducted a study on ML estimators for reliability growth models, confirmed this shortcoming) (Ref 9:20).

The Gompertz (Virene) Model

E.P. Virene, of the Boeing Company, used the Gompertz equation as a general reliability growth model in 1968 (Ref 56:265-270). The model fits a reliability growth curve through cumulated percentage points of reliability plotted against time. The model has been used to determine reliability growth in government programs such as the Lunar Orbiter Spacecraft and the Blue Scout Launch Vehicle (Ref 56:265).

The Model. The form of the Gompertz equation is

$$R(t) = ab^{c^t}, \quad t \geq 0 \qquad (3.34)$$

where

$R(t)$ = system reliability at time t

a = the upper limit of reliability as t approaches infinity

b = the base parameter

c = the shape parameter

t = the test time such as cycles or average operating time per unit equipment age

When using this equation, the parameters a and b must be between zero and one. In addition, parameter c must be between zero and one if reliability growth is being modeled.

There are also assumptions concerning the variable t (Ref 33:D-59). The values of t must be in the same units for each data point observed, and the intervals of t must be

equal. It is also assumed that the set of data points used to estimate the parameters are divided into three equally sized groups.

Parameter Estimation. Consider data used to estimate parameters in the logarithmic form (base 10). Then, Eq (3.34) becomes

$$\log(R) = \log(a) + c^t \log(b) \tag{3.35}$$

The logarithms of the percentage points of reliability are summed up in each of the three groups to give

$$S_1 = \sum_{t=0}^{n-1} \log(R_t) = n\log(a) + \left[\sum_{t=0}^{n-1} c^t\right]\log(b) \tag{3.36}$$

$$S_2 = \sum_{t=n}^{2n-1} \log(R_t) = n\log(a) + \left[\sum_{t=0}^{2n-1} c^t\right]\log(b) \tag{3.37}$$

$$S_3 = \sum_{t=2n}^{3n-1} \log(R_t) = n\log(a) + \left[\sum_{t=2n}^{3n-1} c^t\right]\log(b) \tag{3.38}$$

where n = the number of points in each group and R_t = the percentage reliability for the t^{th} data point. The first data point in group one is always transformed to $t = 0$.

Subtraction yields

$$S_1 - S_2 = \left[\sum_{t=0}^{n-1} c^t - \sum_{t=n}^{2n-1} c^t\right]\log(b) \tag{3.39}$$

and

$$S_2 - S_3 = \left[\sum_{t=n}^{2n-1} c^t - \sum_{t=2n}^{3n-1} c^t\right]\log(b) \tag{3.40}$$

Taking the ratio of Eqs (3.39) to (3.40) gives

$$\frac{S_1 - S_2}{S_2 - S_3} = \frac{\sum_{t=0}^{n-1} c^t - \sum_{t=n}^{2n-1} c^t}{\sum_{t=n}^{2n-1} c^t - \sum_{t=2n}^{3n-1} c^t} \tag{3.41}$$

By changing the limits of the summation in the denominator of Eq (3.41), c^n can be factored out to give

$$\frac{S_1 - S_2}{S_2 - S_3} = \frac{\sum_{t=0}^{n-1} c^t - \sum_{t=n}^{2n-1} c^t}{c^n\left[\sum_{t=0}^{n-1} c^t - \sum_{t=n}^{2n-1} c^t\right]} = \frac{1}{c^n} \tag{3.42}$$

Therefore,

$$c = \left[\frac{S_2 - S_3}{S_1 - S_2}\right]^{\frac{1}{n}} \tag{3.43}$$

which is the estimate for parameter c.

Rearranging Eq (3.39) gives

$$\log(b) = \frac{S_1 - S_2}{\sum_{t=0}^{n-1} c^t - \sum_{t=n}^{2n-1} c^t} = \frac{S_1 - S_2}{\sum_{t=0}^{n-1} c^t(1-c^n)} \tag{3.44}$$

Next, a trick is used to get an expression to substitute into Eq (3.44) for the summation term. Since

$$\sum_{t=0}^{n-1} c^t = 1 + c + c^2 + c^3 + \ldots + c^{n-1} \tag{3.45}$$

and

$$\left[\sum_{t=0}^{n-1} c^t\right] c = c + c^2 + c^3 + \ldots + c^n \tag{3.46}$$

Then by subtracting Eq (3.46) from Eq (3.45), the following equation results

$$\sum_{t=0}^{n-1} c^t - \left[\sum_{t=0}^{n-1} c^t\right] c = 1 - c^n \tag{3.47}$$

Factoring and solving for the first term gives

$$\sum_{t=0}^{n-1} c^t = \frac{1 - c^n}{1 - c} \tag{3.48}$$

Substituting Eq (3.48) into Eq (3.44) for the summation term gives the expression

$$\log(b) = \frac{(S_1 - S_2)(1 - c)}{(1 - c^n)^2} \tag{3.49}$$

which is the estimate for parameter b.

The final parameter, a, is estimated by making substitutions from Eq (3.48) and Eq (3.49) into Eq (3.36) and solving for log(a). The resulting equation is

$$\log(a) = \frac{1}{n}\left[S_1 - \frac{S_1 - S_2}{1 - c^n}\right] \tag{3.50}$$

Example. Suppose the data in Table IV summarize the results of 15 launches of a missile system. Since no successful launches occurred until the fourth launch, no growth was assumed and group one ($t = 0$) starts with launch number 4 (Ref 56:268). The sum of the groups are

$$S_1 = 1.398 + 1.301 + 1.223 + 1.456 = 5.378 \tag{3.51}$$

$$S_2 = 1.574 + 1.647 + 1.699 + 1.736 = 6.656 \tag{3.52}$$

TABLE IV

Missile Data for Gompertz Estimation

Launch Number	Success/ Failure	t	Reliability (% R)	log(R)
1	F	-	---	---
2	F	-	---	---
3	F	-	---	---
4	S	0	25.0	1.398
5	F	1	20.0	1.301
6	F	2	16.7	1.223
7	S	3	28.6	1.456
8	S	4	37.5	1.574
9	S	5	44.4	1.647
10	S	6	50.0	1.699
11	S	7	54.5	1.736
12	S	8	58.2	1.765
13	S	9	61.7	1.790
14	S	10	64.2	1.807
15	S	11	66.7	1.824

$$S_3 = 1.765 + 1.790 + 1.807 + 1.824 = 7.186 \qquad (3.53)$$

From Eq (3.43)

$$c = \left[\frac{S_2 - S_3}{S_1 - S_2}\right]^{\frac{1}{n}} = \left[\frac{6.656 - 7.186}{5.378 - 6.656}\right]^{\frac{1}{4}} = .802 \qquad (3.54)$$

and from Eq (3.50)

$$\log(a) = \frac{1}{n}\left[S_1 - \frac{S_1 - S_2}{1 - c^n}\right] = 1.890 \qquad (3.55)$$

Therefore, a = 77.6 = the upper reliability limit.

From Eq (3.49)

$$\log(b) = \frac{(S_1 - S_2)(1 - c)}{(1 - c^n)^2} = -.740 \qquad (3.56)$$

Therefore

$$b = .182$$

and the reliability equation is

$$R(t) = 77.6(.182)^{.802^t}, \quad t \geq 0 \qquad (3.57)$$

From this equation, the reliability for any past or future point can be computed. It should be cautioned, however, that in this case t = 0 is considered the fourth launch and, subsequently, R(t) is the reliability for the $(t + 4)^{th}$ launch.

Virene also cautions that when using this model, a goodness-of-fit by comparison should be made as a check for suitability. The argument is, if the model compares favorably with existing data, then projections made with the model will also be good (Ref 56:266).

The Bonis Model

Austin J. Bonis, of the Rochester Institute of Technology, used a modified exponential function to model

reliability growth of one shot devices (Ref 10:181-185). The model is not only capable of producing reliability growth curves from test data, it can also produce a "target" growth curve for a test program. The target growth curve can be used to monitor progress towards the reliability goal set for the program.

The Model. The mathematical form of the Bonis model is

$$R_k = R_\infty - QB^{k-1} \tag{3.58}$$

where

R_k = the reliability on the k^{th} test

R_∞ = the ultimate value of reliability that could be attained if k were allowed to increase without limit

Q = the initial unreliability or probability of failure before the test begins

B = the improvement factor

k = the number of the stage

The constant, R_∞, will shift the curve up or down by a constant amount.

A somewhat limiting factor is that the improvement factor, B, is constant throughout the process. This means whatever improvement factor is estimated, is assumed to be the constant improvement factor for the entire test program.

Parameter Estimation. The parameters in this model are estimated by solving the equation that represents the first three stages (Ref 10:183). The equations for the first three stages are

$$R_1 = R_\infty - QB^0 \tag{3.59}$$

$$R_2 = R_\infty - QB^1 \tag{3.60}$$

$$R_3 = R_\infty - QB^2 \tag{3.61}$$

Subtraction yields

$$R_1 - R_2 = -Q(B^0 - B^1) = -Q(1 - B) \tag{3.62}$$

and

$$R_2 - R_3 = -Q(B^1 - B^2) = -QB(1 - B) \tag{3.63}$$

By dividing Eq (3.62) into Eq (3.63)

$$B = \frac{R_3 - R_2}{R_2 - R_1} \tag{3.64}$$

which is the estimate for B. Also, solving for Q in Eq (3.62) gives

$$Q = \frac{R_2 - R_1}{1 - B} \tag{3.65}$$

Finally, R_∞ is determined by solving Eq (3.59)

$$R_\infty = R_1 + Q \tag{3.66}$$

When using Eqs (3.64), (3.65), and (3.66) to estimate the parameters, k is assumed to be a consistent measure of time. Also, the data points used in the estimating equation should either be the first three or three consistent groups with the same number of data points in each group.

<u>Example</u>. Suppose a missile development program consists of testing 10 missiles at each stage for 10 stages.

The desired reliability at the end of the program is 90 percent successes. The first three stages resulted in two, six, and seven successes respectively. Management wants to know if the present development will result in a 90 percent success ratio at the end of the program based on the results of the first three stages.

In this case, $R_1 = 2/10 = .2$, $R_2 = 6/10 = .6$, and $R_3 = 7/10 = .7$, which are the estimated reliability levels of the first three stages based on the results of the test. Using Eqs (3.64), (3.65), and (3.66), the parameters are estimated as

$$B = \frac{R_3 - R_2}{R_2 - R_1} = \frac{.7 - .6}{.6 - .2} = \frac{.1}{.4} = .25 \qquad (3.67)$$

$$Q = \frac{R_2 - R_1}{1 - B} = \frac{.6 - .2}{1 - .25} = \frac{.4}{.75} = .53 \qquad (3.68)$$

$$R_\infty = R_1 + Q = .2 + .53 = .73 \qquad (3.69)$$

The answer to the question posed by the management is clear before actually calculating R_{10}, because R_∞ is the highest reliability that can be hoped for in the present program. However, R_{10} can be easily calculated by substituting in the appropriate values. Thus,

$$R_{10} = .73 - .53(.25)^{10} = .729 \qquad (3.70)$$

which is short of the reliability goal. Of course, management can now change the development program and start the estimation all over again (Ref 10:181-183).

Other Models

The Weiss Model. Herbert K. Weiss, of Northrup Aircraft, Inc., stated that many complex systems, especially those involving numerous electronic components, are subject to failures with operating time that follow a Poisson-type distribution (Ref 57:532). Assuming the mean time to failure (MTTF) changes by a constant percentage, the model has the form

$$T(i) = Ae^{ci} \tag{3.71}$$

where T(i) is the (MTTF) for the i^{th} trial or stage, and A and c are parameters to be estimated.

The assumptions for using the model are, first, a simple system is assumed where the MTTF is believed to change at a rate that is unknown on successive trials. In addition, failures are assumed to occur according to a Poisson process. Lastly, a series of trials are assumed with the time to failure on each trial being recorded.

Weiss develops estimates for A and c by the method of maximum likelihood. The ML estimators must be computed in an iterative process where an initial value is assumed for c, and then two values for A are computed from the ML equations. The process is repeated until the two values calculated for A are equal (within a specified tolerance).

A nice characteristic of this model is the ability to include other functions for T(i). For example, it can be simplified to T(i) = A if there is no change in the MTTF with each successive trial number.

<u>The Wolman Model</u>. W.W. Wolman considered a model which takes into account three distinct outcomes of an experiment--success, inherent failure, or assignable cause failure (Ref 59:144-160). Inherent failures are considered random, while assignable cause failures are those that are attributed to a design weakness or, therefore, correctable. Wolman uses a Markov-chain approach to derive the model.

The Wolman model suffers from three weaknesses. First, it requires that the number of assignable cause failures be known in advance. Second, Wolman has no procedure defined for estimation of parameters in the model. Finally, it lacks the ability to make projection about future reliability (Ref 9:41-42).

<u>Barlow and Scheuer Model</u>. Richard E. Barlow and Ernest M. Scheuer, of the University of California at Berkeley and the Rand Corporation, also considered a trinomial reliability growth model (success, inherent failure, and assignable cause failure, as with the Wolman model) (Ref 5:53-60). Unlike Wolman's probabilistic model, theirs is a nonparametric-statistical model. As with the Wolman model, inherent failures are defined as those associated with a system at the state-of-the-art. Assignable cause failures are those which can be corrected by equipment or operational modifications. The reliability of the system at the i^{th} stage is defined as

$$R_i = 1 - q_o - q_i \tag{3.72}$$

where q_o is the probability of an inherent failure, and q_i is the probability of an assignable cause failure at the i^{th} stage. The parameters q_o and q_i are estimated by the method of maximum likelihood (ML).

The assumptions of the Barlow-Scheuer model are that there will be k stages of testing. At each stage, a certain number of test trials are conducted which may be fixed in advance or randomly. The results of the tests from one stage are used to make improvements to the system at future stages. In addition, any changes that are made to the system are assumed to increase the reliability of the system. One of the problems with the model is that Barlow and Scheuer provide no functional relationship for the reliability growth process. This means it is not possible to make reliability projections (Ref 9:40).

The Gross and Kamins Models. Gross and Kamins, of the Rand Corporation, investigated four generalized reliability growth models (Ref 27:406-416). One of the models was a generalized form of the Lloyd and Lipow model. The other models were variations of a model very much like Lloyd and Lipow's. None of the models emerged as a clear choice, and Gross and Kamins suggested that yet another variation, an "adaptive model," would yield good results (Ref 4:18).

Gross and Kamins present several tables and graphs which show, in a comparative way, the results of their efforts. In addition, their concluding remarks point out the significant results of the research in a list of nine

points.

The Pollock Model (Bayesian). Stephen M. Pollock introduced a reliability growth model that incorporates the Bayesian concept of prior information (Ref 49:187-198). Pollock's main objectives were to gain inference on estimation (the present value of reliability) and projection (the reliability at some future time), with or without continued application of the correction or growth processes (Ref 58: 472-475). In addition, he includes both continuous and discrete cases.

Pollock provides a detailed discussion and rigorous mathematical presentation of Bayesian methods and how they apply to reliability growth. He has even taken into account the notion that reliability may decrease rather than increase as the development process is administered (Ref 4:30). The modeling technique presented by Pollock appears to have good monitoring potential and also good projection capability.

The Singpurwalla Time Series Method. Nozer D. Singpurwalla, of George Washington University, has done work in reliability using time series analysis techniques. In 1975, he proposed a method for forecasting reliability growth or deterioration based on a time series analysis approach (Ref 53:1-14). He placed particular emphasis on the fact that the time series process he discusses will measure both an increase and a decrease in failure rate.

The assumptions of the model are, first, outcomes of each test are determined to be either successes or

failures. In other words, at the end of the j^{th} stage, N_j independent tests have been completed with r_j successes. If P_j denotes the reliability at the end of the j^{th} stage, then r_j is binomially distributed with parameters N_j and P_j.

Singpurwalla offers two techniques for directly estimating the parameters N_j and P_j. First, he discusses the method of maximum likelihood. Then, because it is often desirable to modify estimators based on prior information, he suggests using Bayesian estimators. In addition, he provides a discussion on how to work with transformations of the estimators.

The time series analysis Singpurwalla suggests using is the Box and Jenkins method (Ref 11). This is an autoregressive integrated moving average (ARIMA) model, and the technique is to model the data and then analyze the residuals. The determination of a positive trend term would imply reliability growth is present, while a negative trend term would indicate deterioration of reliability.

Singpurwalla emphasizes two advantages to his approach. First, his method does not require a particular reliability growth model be specified. This implies more flexibility for the analyst. Next, the method allows the analyst to incorporate deterministic inputs such as engineering judgments or managerial interventions (Ref 53:2).

A disadvantage to the approach is that it requires data in a large number of stages. This is necessary so trends in the residuals can be determined.

IV. Dormancy and Dormant Reliability

Almost any system conceived will spend a portion of its life in a non-operating state. In a non-operating state, the electrical or mechanical stresses normally associated with an activated state are not present, but this does not mean that all stresses to the system are absent (Ref 32:16). Other stresses may be at work--the environment, transportation, or handling, to name a few.

Reliability, of course, is state dependent. If the true reliability of a system is to be known, the nature of each state of the system must be taken into account. However, this has not always been done in the past. For example, Rocco F. Ficchi, an engineer for Radio Corporation of America, stated:

> It is generally assumed that parts and equipment subject to zero electrical stress had zero failures. This has been shown to be untrue [Ref 24:42].

Engineers now realize that reliability in a non-operating state is very important. For example, certain of our strategic weapon systems (Minuteman missile system, for one) spend all their life in non-operating states. In addition, new weapon systems, like the cruise missile, are being built which will spend most of their life in a non-operating condition (Ref 32:7). This has led to an increasing interest in the reliability of non-operating systems (Ref 32:11).

Definitions

Non-operating State. A non-operating state is actually a set of states that can be divided into several subsets. Examples of these subsets are: storage, inherent dormancy (totally non-operating but not in storage), and operationally ready storage (awaiting operational use with some subsystems energized) (Ref 32:17). However, in this study it is not purposeful to differentiate between the different non-operating states. Therefore, terminology which has been more precisely defined elsewhere will not be used here. Instead, the following definitions will be used to refer to a general non-operating state.

Dormancy. This is a state where a system experiences either no operational stress, or very low levels of operational stress (Ref 24:25). Dormancy can include such states as non-operating portions of alert, transportation and handling, and launcher carriage as examples (Ref 32:18).

Dormant Reliability. Reliability is defined as the probability that failure does not occur before a specified time (see Eq 2.2). Consistent with this definition, dormant reliability will be referred to as the probability that a system in a dormant state does not fail prior to a specified time. In other words, dormant reliability is the reliability of a system in a dormant state.

Methods of Analysis

Dormant reliability analysis has been identified as

falling into three broad categories (Ref 32:27). These categories are: 1) parts count and stress analysis; 2) failure rate modification factors (K factors); and 3) testing (accelerated and field testing). Each of these methods have good and bad points, which helps indicate which to use in a given situation. In the following paragraphs, each method will be briefly described stressing important points of each method.

Parts Count and Stress Analysis Method. This technique assumes that system reliability can be calculated if the reliabilities of each component are known (Ref 32:27). The failure rates of each component are determined and then summed together to get the system failure rate. The component failure rates are often obtained from tables like the MIL-HDBK-217C. These tables are the results of both empirical data and laboratory testing on previous equipment (Ref 39).

This method has been used extensively in the design phase of systems or subsystems (Ref 32:27). It is particularly useful as a comparative evaluation, but it has not been conclusively checked against empirical data from dormant systems. For this reason, it is not likely that the operational analyst would use this procedure for dormant reliability prediction (Ref 32:27).

Failure Rate Modification Factors. Adjustments to failure rates which account for varying stresses, like the environment, are generally called K factors (Ref 32:28).

The idea of K factors is to modify the failure rate of a system to achieve more accurate predictions. K factors are usually developed from field data or laboratory testing with systems or components (Ref 32:28-29).

It is important to stress that K factors must be used with caution. Unless specifically validated against empirical results, K factors could simply be wrong. However, they have been used with good results in certain situations (Ref 32:60).

Testing. Testing is probably the most desirable method of analyzing dormant reliability; however, it is also the most expensive. The two factors that are primary considerations in any testing program are sample size and time (Ref 32:30). Of course, both of these factors can be measured in dollars. Many times, a limited budget makes it impossible to use testing to analyze a dormant system.

Testing may be done in either real time or in an accelerated way (Ref 32:30). Accelerated testing can be used if the predominant failure mode is known. For example, if seasonal humidity cycles (changes in relative humidity with the seasons) are known to cause a seal to fail in a hydraulic system, then the humidity cycles can be accelerated in the laboratory to simulate real time data. This has been shown to be a very effective way of determining failure rates (Ref 32:31). However, it is important to remember that it is not exactly the same as real time data. There could be unforeseen failure mechanisms that would not show up in the

accelerated test data. For this reason, accelerated test data should be taken in the proper context.

Real time testing or surveillance testing, on the other hand, uses data gathered from actual environmental conditions (Ref 32:32). Given the time and the appropriate sample size, this method can be a very accurate way to determine dormant reliability. However, the failure mechanisms in dormant systems are often very slow to surface. For instance, it could take years of dormancy before a decrease in reliability of an electronic circuit would be noticed (Ref 21:14). For this reason surveillance testing is not always practical (Ref 32:33). Also, it is generally very expensive to administer a surveillance test which is another reason why other methods of testing are often chosen.

Dormancy Modeling

Dormant systems are generally characterized by changing failure rates that are increasing over time. The problem with modeling most dormant systems is usually due to the slow rate at which failures occur. The failure rate may change only minutely in the first several years of an equipment's life. The variance in a set of data is, at times, large enough to completely mask dormant reliability. For example, Dickhaut and Dudley did a study of the long-term storage of microelectronic components, and were completely unable to make predictions based on data that were available (Ref 21:40-44). The problems encountered in

the study were typified by data with too much variability to provide statistical trends. In addition to very slow changes in failure rates, dormant systems also have quite small initial failure rates in many cases. Failure rates of microelectronic devices, for example, can be as low as 1×10^{-9} units (Ref 8:3-5 to 3-324). Again, this means variability in measurements can be a problem when analyzing data. Even though problems with dormant data exist, attempts have been made to model dormant systems, particularly during the design phase.

Dormant reliability modeling in the past has been done most often in conjunction with life-cycle modeling in the design phase of systems (Refs 1; 7; 8; 20; 21; 23; 34; 35). Life-cycle modeling takes into account the reliability of each phase of a system's life (Ref 32:C-1). Dormancy, under this concept, consists of periods of time in the life of a system where the system is in a dormant state. The failure rate modification method is used to determine a failure rate for the specified time period and the environmental conditions that are assumed for that time period. This failure rate is then used as the parameter in the negative exponential model and the reliability can be calculated. This technique is used under the concept of "periodic testing" or "no testing" (Ref 32:C-2). Under the "periodic testing" concept, the system is tested at certain intervals, and if necessary, repaired. This brings the reliability level back to the original level or very nearly so. It

should be noted that no evidence was found (in the form of technical reports) of this technique being validated with empirical data.

The literature search on dormant reliability produced no widely accepted modeling techniques on operational systems. Reasons for this have not been specifically defined. However, one could speculate that the cost of gathering empirical data has been judged to outweigh the information received from the data. Also, the military may be one of only a few organizations with a real concern about dormant reliability. After all, private sector businesses are generally concerned with producing and selling, not with storing.

V. The Monte Carlo Experiments

The fact that both reliability growth and dormant reliability are characterized by changing failure rates was the motivation for the Monte Carlo experiments. Certain reliability growth models use the method of fitting a curve to a set of data. From the curves, which smooth the data, reliability estimations can be inferred. Some reliability growth models are capable of modeling data that has either a decreasing failure rate (reliability growth) or an increasing failure rate (as with dormant reliability). With some models, the original assumptions may have to be changed to handle increasing failure rates, but this problem can usually be overcome. For example, the parameter (α) in the Duane model is assumed to be between zero and one if reliability growth is present. However, by changing this assumption and allowing α to take on values greater than one, an increasing failure rate can be modeled. Therefore, models like the Duane can be considered candidates for dormant reliability modeling.

The purpose of the Monte Carlo experiments was to observe results of modeling attempts, i.e. modeling dormant data with reliability growth models. The results are important because they potentially provide a new methodology for analysts who work with dormant data. There is already

a good variety of reliability growth models in existence, and the methodology behind most of them is well developed. Any evidence that shows reliability growth models to be useful in modeling dormancy would provide alternative methodologies to solve the problem.

Monte Carlo techniques were chosen to generate data for the experiments for two reasons. First, no data from a dormant system could be found that was suitable for the experiments. Only one good set of data was obtained. This data was the result of field tests in the AGM 65 A/B (Maverick) missile system. The problem with the data was that it showed too small an increase in the failure rate to be useful in a comparative analysis and, therefore, was not used.

The second and most important reason Monte Carlo methods were used was to have control over the underlying failure distributions of the system. Since the underlying distributions were known, the true system reliability could be calculated and used as a comparative basis for the "goodness-of-fit" measures. In addition, the underlying distribution can be easily changed for future experimentation.

Models Tested

Three reliability growth models were selected for the experiments. They are the Gompertz model, the Bonis model, and the Duane model. There were three primary factors considered in the selection of these models. First,

any model chosen had to have the ability to predict future reliability (this is an extrapolation of the estimated function). All three models met this requirement. Second, the underlying assumption in the models chosen had to be such that dormant data could be modeled. Again, the three models chosen met the requirement. Lastly, because computer time was a constraint, it was desirable to select models in which the parameters were relatively easy to calculate. The design of the simulation called for the parameters to be estimated 1200 times for each model. Therefore, models that require iterative parameter estimation techniques were ruled out. An additional factor that was considered was the popularity of the models in the military. These three models have been used extensively in military programs, particularly the Duane model. For these reasons, it was decided that the three models chosen would represent a good set for initial experimentation.

The Gompertz Model. This model has three parameters which are estimated from explicit equations using time dependent data. Mathematically, this model is represented by

$$R(t) = ab^{c^t} \qquad (5.1)$$

This same form was used to model the dormant data with one exception. The parameter a is defined as the upper value of reliability of a system (see the discussion of this model in Chapter III). In the simulation, the assumption was made that all systems that were placed in storage at the beginning

of the testing program were nonfailed (good systems). Since parameter a represents the highest attainable reliability, this implies that a = 1, and the functional form of the model reduces to

$$R(t) = b^{c^t} \qquad (5.2)$$

where

$R(t)$ = the probability the system will be a success at time t

b = the base parameter

c = the shape parameter

t = time

The parameters b and c were estimated using Eqs (3.49) and (3.43) respectively. To make Eq (5.2) a valid measure of reliability, the parameter b must be restricted to values between zero and one. Also, if an increasing failure rate is assumed, c must be strictly greater than one. If these conditions were not met in the simulation, the model was considered invalid. The primary cause of not meeting the restrictions was not enough consistent data.

The Bonis Model. Assumptions concerning the data when using the Bonis model are similar to those when using the Gompertz model. The data must be used in three consecutive groups (see the discussion on the Bonis model in Chapter III). The form of the Bonis model is

$$R_k = R_\infty - QB^{k-1} \qquad (5.3)$$

R_∞ represents the upper limit of reliability as k is increased without bound (see the discussion of this model in Chapter III). This parameter is very similar to the parameter a in the Gompertz equation. Also like a in the Gompertz equation, R_∞ is assumed to be one, because all nonfailed systems are assumed at the start of the test program.

The other two parameters, B and Q, are estimated with Eqs (3.64) and (3.65) respectively. Since Q is defined as the initial unreliability, its value was restricted to between zero and one. In addition, B must be strictly greater than one if an increasing failure rate is to be represented. Once again, if these assumptions were not met, the model was considered invalid.

A particular problem with the Bonis model is that it has no inflection point, and R_k will decrease without bound when used to model dormant data. In other words, once the reliability starts a downward trend, it continues downward without bound as k is increased. Extrapolated values for reliability obtained from this model are invalid after a certain point. However, there is still a rationale for using the model. For instance, many times an analyst may be interested in finding the time (k) where the reliability level of interest is relatively high (generally above the inflection point in the true reliability curve); the Bonis model should provide acceptable results.

<u>The Duane Model</u>. The Duane model uses two methods of parameter estimation--the graphical technique and least

squares estimators. The least squares estimators are a mathematical representation of the graphical technique. Therefore, least squares was chosen to estimate the parameters. The Duane model is given as

$$N(t) = Kt^{1-\alpha} \tag{5.4}$$

where

$N(t)$ = the cumulative number of failures

α = the decreasing reliability rate constant

K = a constant that represents the cumulative failure rate at $t = 0$

t = time

Dividing Eq (5.4) by t and taking logarithms gives Eq (3.4), which is used to estimate the parameters α and K.

Once the two parameters have been estimated, they are used in Eq (3.3) to obtain instantaneous failure rates for values t. The instantaneous failure rates are transformed into reliabilities using the negative exponential model.

The rationale for using the Duane model is that with reliability growth, the cumulative failure rate was a decreasing function of time. With dormant reliability, the opposite occurs; the cumulative failure rate increases with time.

Simulation Methodology

The methodology was to use Monte Carlo techniques to generate sets of dormant data which were subsequently modeled with reliability growth models. After the data were modeled

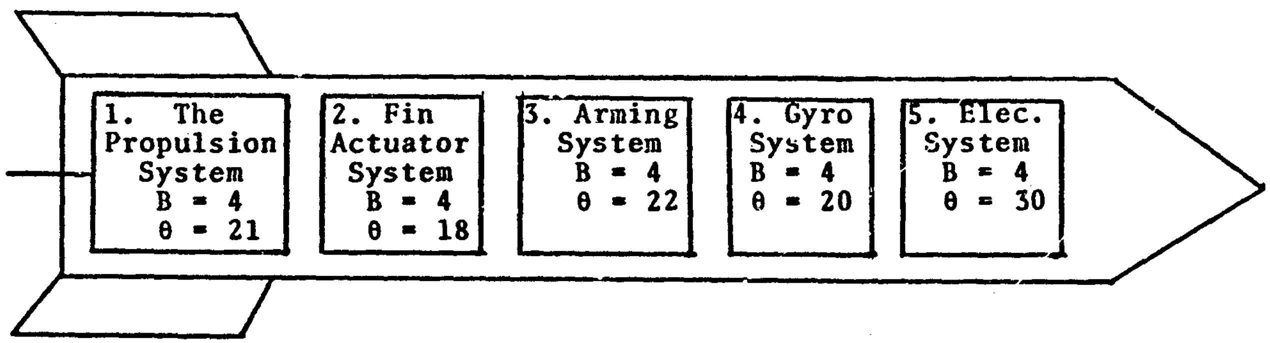
(parameters estimated), reliabilities were estimated using the functional relationships associated with each model. The estimated reliabilities were then compared with the true reliabilities based on the underlying failure distributions which were used to generate the data. The result was a set of relative measures (which can be thought of as relative goodness-of-fit measures) of the model against the true system reliability.

The System. In order to add a sense of realism, a fictitious missile system was contrived which will be referred to as hypothetical missile system one (HM-1). The HM-1 consists of five independent subsystems in series: 1) the propulsion system; 2) the servo-mechanical fin actuator; 3) the arming system; 4) the gyro; and 5) the electronic control system. Each subsystem has a separate failure distribution associated with it. The Weibull failure distribution

$$f(t) = \frac{Bt^{B-1}}{\theta^B} \text{EXP} - \left[\frac{t}{\theta}\right]^B, \quad t \geq 0 \qquad (5.5)$$

was assumed to be the underlying distribution in each subsystem. In this distribution, B is the shape parameter (governs the shape of the density function), and θ is the scale parameter (stretches the density distribution out) (Ref 30:22).

The five subsystems and their associated parameter values are shown in Fig. 2. Of course, the knowledgeable



1. All subsystems Weibull distributed
2. Location parameter equals zero
3. B = shape parameter
4. θ = scale parameter

Fig 2. The HM-1

reader will realize that the five separate distributions could have been combined into a single Weibull failure distribution because the shape parameter, B, is identical for each subsystem. However, the computer simulation was written with the intent of making future changes to the failure distributions of the subsystems. Therefore, even though one distribution could have modeled the entire system in this case, separate distributions were used to maintain generality.

In each subsystem, the shape parameter, B, was given a value of four. Even though the value of B was arbitrary, the intent was to closely approximate the normal distribution (Ref 30:294). In other words, it was assumed that the failures of each subsystem were normally distributed, but were approximated by the Weibull distribution with B = 4. The scale parameters were contrived so the system would decrease in reliability from one to approximately zero as time (t) was increased from zero to 20. The reasoning behind this was to force the models to contend with a full range of reliabilities and see how well the predictions would be under this contention.

Data Generation. Monte Carlo techniques were used to generate data for the modeling experiments. The data were generated as either a missile failure or success. The steps for determining the successes or failures in a set of data are as follows:

1. The reliability of each subsystem was calculated

for the specified value of time (t).

2. Generate a random number for each subsystem and compare it with the subsystem's reliability calculated in Step 1.

3. Since the subsystem reliability is defined as the probability of a success, the subsystem was considered a success if the random number was less than the calculated reliability.

4. Each subsystem is independent and, therefore, all subsystems must be successful if the system is to be a success.

5. Repeat steps two through four until the desired number of missiles per time period have been tested.

6. Increment t, where t = 1,2,3,...,12, and repeat the process until the desired number of years of data have been generated.

The computer used for the simulation was an Apple II microcomputer. The internal pseudorandom number generator, RND( ), was used to generate the random numbers. The validity of the random numbers was checked with two separate serial tests (Ref 41:57-59) done on two separate random number streams of 100,000 random numbers each. In each serial test, 100 chi-square statistics from a frequency test were used as input to another chi-square test (Ref 41:57). The results were values of 5.600 and 6.600 for the chi-square statistics compared with a critical value of 14.684

at α = .10 level of significance. It was concluded that RND( ) was a satisfactory pseudorandom number generator for the experiments.

<u>The Testing Procedure</u>. In the simulation, the testing schedule was varied with the intent of finding which model would work best in a given situation. Also, it was hoped to gain an insight into how much data were needed for one model relative to another.

The first assumption is that a large amount of HM-1 missiles were produced and placed in storage at $t = 0$. Each time period (which could be thought of as years), a specified number of missiles are brought out of storage and tested. For example, in the first simulation run 50 missiles per time period were brought out of storage and tested. This was the first testing policy used with each model.

After six time periods of testing, the data were modeled and the reliabilities, $R(t)$, for $t = 1,2,3,\ldots,20$ were estimated. This is considered one set of reliabilities (which can be thought of as one dormant reliability curve) based on one set of random numbers. This process was repeated 50 times using different random numbers each time. The result was a set of reliabilities $R_i(t)$ where $i = 1,2,3,\ldots,50$ and $t = 1,2,3,\ldots,20$. Using the central limit theorem (Ref 37:252), the mean and variance for the $R_i(t)$'s were calculated for $t = 1,2,3,\ldots, 20$ where the mean is

$$\bar{R}(t) = \frac{1}{50} \sum_{i=1}^{50} R_i(t) \qquad (5.6)$$

and the sample variance is

$$S(t) = \frac{\sum_{i=1}^{50} [R_i(t) - \bar{R}(t)]^2}{n-1} \qquad (5.7)$$

The mean, $\bar{R}(t)$, was the dormant reliability curve that was used for the comparison with the true system reliability curve. This will be discussed in further detail later.

The next logical step was to add more data. This was first done by leaving the same test program in effect (50 missiles per time period tested), but instead of using only six time periods of data, nine time periods of data were used. Increments of three time periods were chosen for increases in data because the Gompertz and Bonis models both require data in three equally-sized groups. Finally, 12 time periods of data were used to make estimates with the same test program in effect. This resulted in three sets of $\bar{R}(t)$'s or three average dormant reliability curves, one for six, one for nine, and one for twelve time periods of data.

The preceding procedure was considered to be one simulation run based on one test program. The final step was to change the test program from 50 missiles per time period tested to 100 missiles per time period tested in increments of 10. This meant that six different simulation runs were made on each model with each run using a different test program. The idea was to start with a minimal amount

of data, increase it by increments, and observe the behavior of the dormant reliability curves as more data were used.

Relative Measures

In order to include a numerical comparison of how well each model did under a given test program, three relative measures were used. These measures (M1, M2, and M3) can be thought of as relative goodness-of-fit measures and are similar to ones used by Toke Jayachandran in his study on reliability growth (Refs 28; 29).

Measure One (M1). This measure is simply the sum of the discrepancy between the true reliabilities, $R^*(t)$, and the average reliabilities, $\bar{R}(t)$, for a given test program. Mathematically, the measure is

$$M1 = \sum_{t=1}^{20} |R^*(t) - \bar{R}(t)| \qquad (5.8)$$

where t is the time periods. M1 measures the total distance the average reliability curve $[\bar{R}(t)]$ is away from the true reliability curve $[R^*(t)]$ at each of the 20 time periods.

Measure Two (M2). This measure is the sum of the squared values of M1 at each of the 20 time periods. This measure is closely associated with the sum of the squared errors which is used in regression analysis. The form of M2 is

$$M2 = \sum_{t=1}^{20} [R^*(t) - \bar{R}(t)]^2 \qquad (5.9)$$

This measure emphasizes average reliabilities that are grossly different from true liabilities.

Measure Three (M3). M3 is defined as the maximum of the squared differenc between the true reliabilities $[R^*(t)]$ and the average estimated reliabilities $[\bar{R}(t)]$. The equation for M3 is

$$M3 = \max_{t}\{[R^*(t) - \bar{R}(t)]^2\} \qquad (5.10)$$

Each of these three measures was calculated for each average dormant reliability curve estimated $[\bar{R}(t)]$. This resulted in 18 sets of measures, one set for the test program where 50 missiles per time period were tested and six time periods of data were used, another set for 50 missiles per time period tested and nine time periods of data used, and so on. The number of missiles tested per time period ranged from 50 to 100 in increments of 10, while the number of time periods of data used was six, nine, or twelve. Thus, a total of 18 sets of relative measures were calculated.

VI. Simulation Results

The results of the Monte Carlo experiments were not as consistent as hoped for. That is, the models did not behave as well as expected. For instance, the predictions from both the Duane and Gompertz models were further from the true reliabilities when the number of time periods of data used was increased from 9 to 12. For the Duane model, this happened only when 90 or 100 missiles per period were tested. However, this phenomenon took place in every case with the Gompertz equation. On the other hand, the Bonis model was generally more consistent. However, the Bonis equation generally required more data to achieve relative measures that were as good as either the Duane or Gompertz model. Also noted was the fact that increasing the number of missiles tested per time period did not make much difference in a model's ability to predict future reliability. On the other hand, increasing the number of time periods of data used to estimate parameters made the reliability predictions significantly better.

The results of the simulation are shown graphically in Fig 3 through Fig 20 at the end of the chapter. Also, the relative goodness-of-fit measures (M1, M2, and M3) follow the graphs in Tables V through X.

Results Using the Gompertz Equation

The most notable inconsistency evident from the relative measures involves the Gompertz equation. The measures indicate better results when nine periods of data were used instead of 12 with every test policy. In other words, the predictions were worse when more data were used. The reason for this inconsistency is not completely understood. However, there were two observations noted that help explain this occurrence.

First, the dormant reliability curves generated by the Gompertz equation are quite sensitive to the base parameter, b, from Eq (5.2). For example, with the shape parameter, c, set equal to 1.4 and b = .99, the reliability at 10 time periods, R(10), is .75. However, when b was changed by .005 to .995, R(10) = .87, a difference of .12. This points to the fact that if b is estimated slightly high, the result will be a dormant reliability curve that overestimates the true reliabilities (shifted to the right). From the graphs, this appears to be the case with the Gompertz equation.

There was also a second trend noticed with the Gompertz model. The shape parameter, c, governs the relative steepness of the reliability curves at the inflection point. The higher the value of c, the steeper the curve will be. Therefore, if the base parameter, b, is estimated too high, the scale parameter, c, can compensate by making

the curve steeper. This will cause the predicted reliability curve to cross or at least converge toward the true reliability curve. This occurred with every testing policy when nine periods of data were used. The average base parameter, b, was always the greatest when nine periods of data were used (.996 to .998), but the shape parameter, c, always compensated by being higher when nine periods of data were used rather than 12 (approximately 1.65 compared to 1.45 for 12 periods of data). The reason for this was not determined, but it resulted in better predictions when nine periods of data were used in every case.

The observations made on the Gompertz model can be visualized in the graphs of Fig 3 through Fig 20. It should be noted, that the data used to estimate b and c was cumulative in nature. This was according to the example suggested by Virene (Ref 56). This means the percent reliability levels used were cumulated percentages of all the data from time t = 0.

Results Using the Bonis Equation

The Bonis model was generally more consistent than either the Duane or Gompertz models. Under every testing policy, the relative measures improved as more data were made available. The problem with the Bonis model lies in the fact that more data were needed to get results that compared equally with the Duane and Gompertz models. This was due in part to the fact that the Bonis model does not

have an inflection point. This means the reliability predictions decrease without bound once the downward trend has started. This makes the model unable to predict reliabilities in the right-hand tail of the reliability curve. This can be seen in Fig 3 through 20. However, the reliability predictions made with the Bonis model appeared to be acceptable for all reliability levels prior to the inflection point of the true reliability curve. In many cases, the fit of the Bonis model appears closer in the early time periods than either the Duane or Gompertz models.

Results Using the Duane Model

The Duane model generally required more data than the Gompertz model, but less than the Bonis model, to get equal results. As with the Gompertz model, however, it also was inconsistent in some cases. For example, as the number of missiles tested per period was increased from 70 to 100, the relative measures increased slightly, indicating the predictions were getting worse. This occurred only when 12 periods of data were used to estimate the parameters. Also, the measures increased when the time periods of data were increased from nine to 12. However, this was only the case when 90 or 100 missiles per period were tested.

Like the Gompertz model, the reason the predictions of the Duane model were getting worse appeared to be a shift in the predicted reliability curve. Unlike the Gompertz model, however, the Duane curves shifted to the left rather

than to the right. One reason for this shift may be in the way the parameters are estimated. From Eq (3.4), it is obvious that the dependent variable, $\ln[N(t)/t]$, is undefined when N(t) (cumulative number of failures) is zero. This means that until a failure occurs, data points cannot be included in the regression equation. During the simulation, the system reliability was high enough in its early life to cause no failures in time periods one through three. At times, there would be no failures until as many as five or six time periods. Of course, the fact that no failures take place early in the system's life is taken into account in some respect by the cumulation of data. However, it must be re-emphasized that once a change in the failure rate is estimated with the Duane model, the change is assumed to be consistent throughout the extrapolation. For example, if there are no failures for the first three time periods and nine time periods of data are used to estimate the parameters, then the last six time periods are used to model the changing failure rate of the system. The model is then extrapolated back to $t = 1$ as well as forward to $t = 20$. The problem is that the failure rate modeled from time period three to time period nine is changing at a faster rate than was the case during the first three time periods. This causes the predictions to be less than would ordinarily be the case. This phenomenon can be visualized from the graphs.

It was also interesting to note how many times there was not enough data for meaningful results. This statistic,

along with the means and standard deviations of the predicted reliability curves for all the models is included in the Appendix.

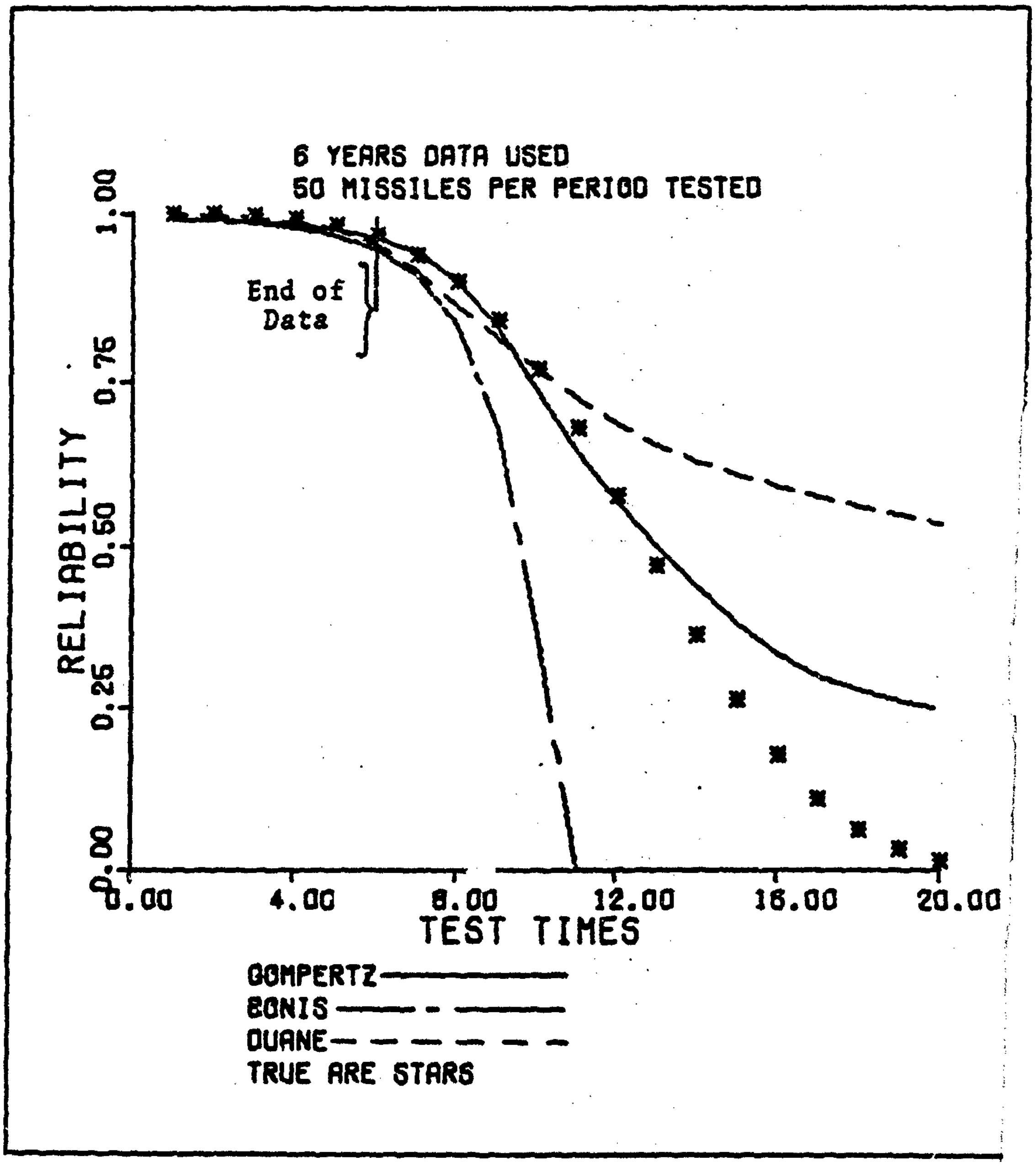


Fig 3. Comparison of Mean Estimated Curves
(6 time periods - 50 missiles)

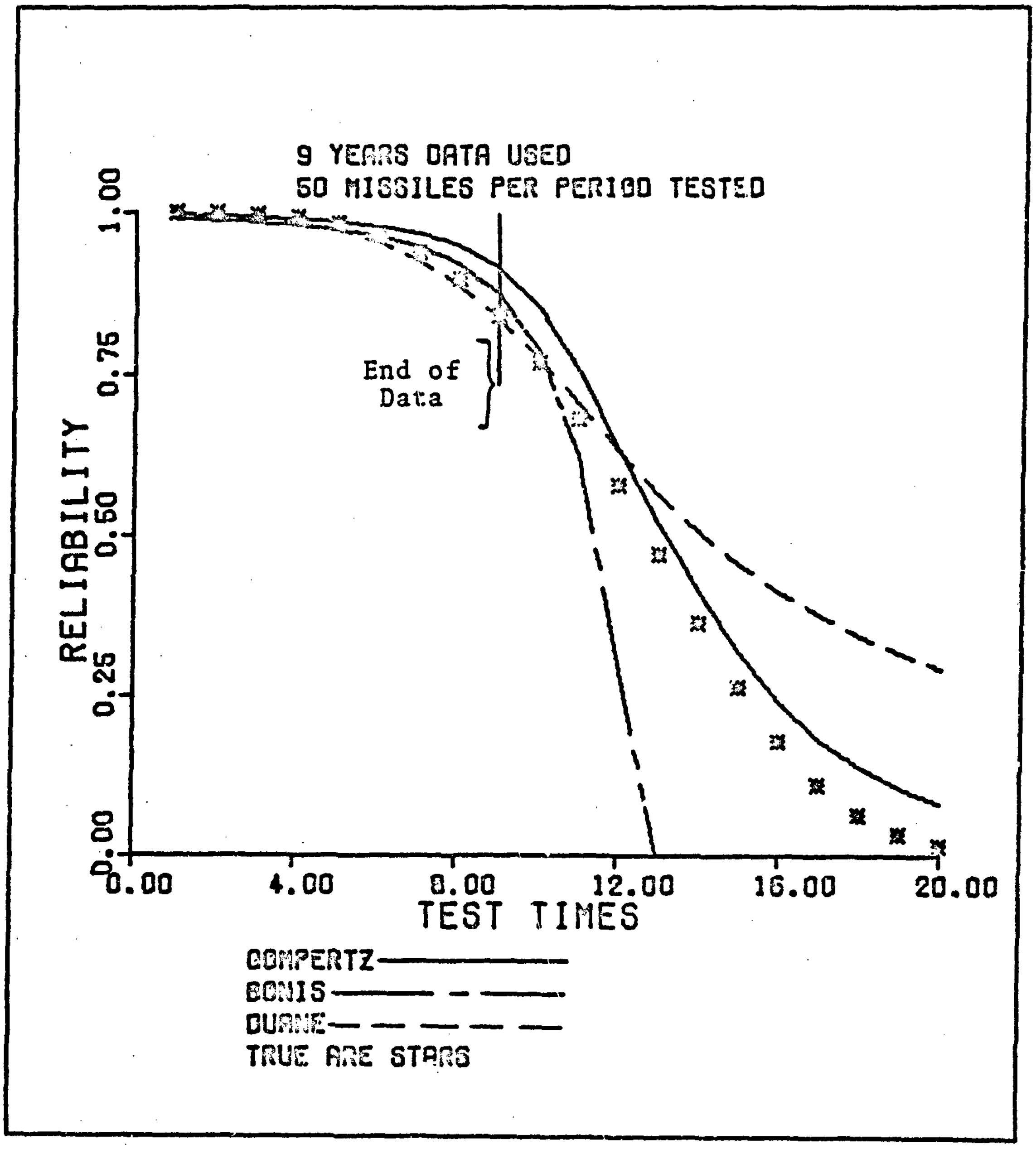


Fig 4. Comparison of Mean Estimated Curves
(9 time periods - 50 missiles)

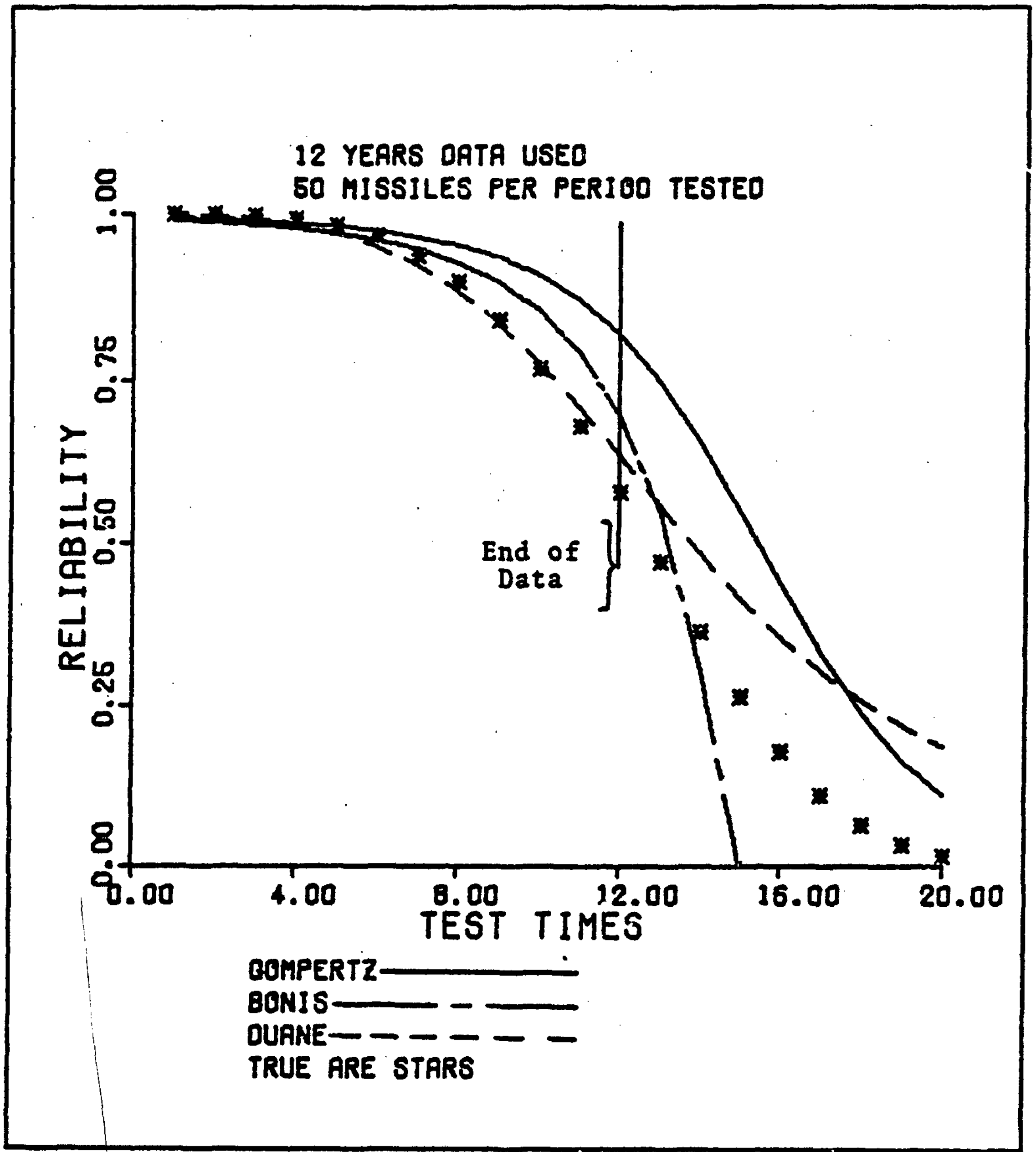


Fig 5. Comparison of Mean Estimated Curves
(12 time periods - 50 missiles)

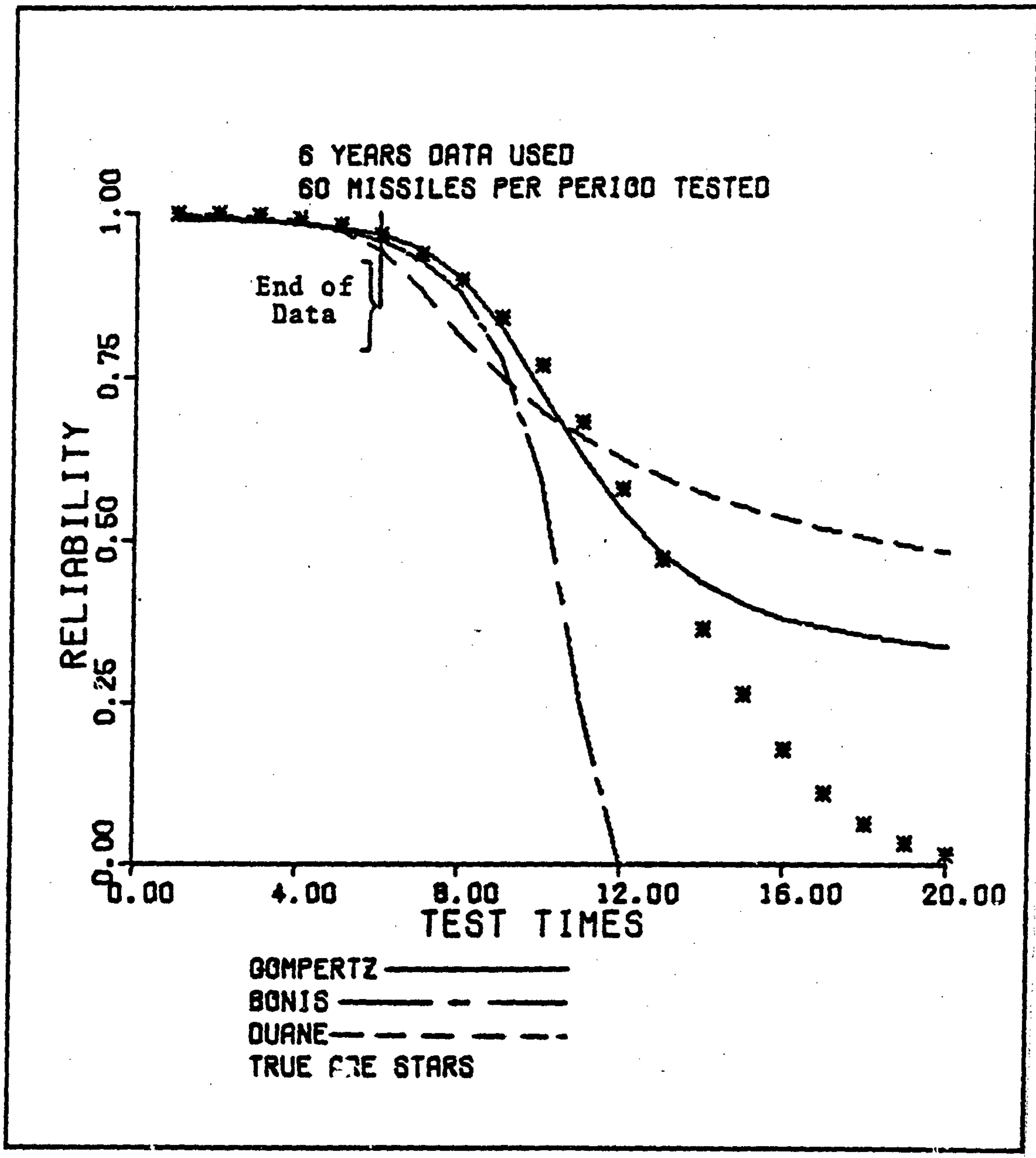


Fig 6. Comparison of Mean Estimated Curves
(6 time periods - 60 missiles)

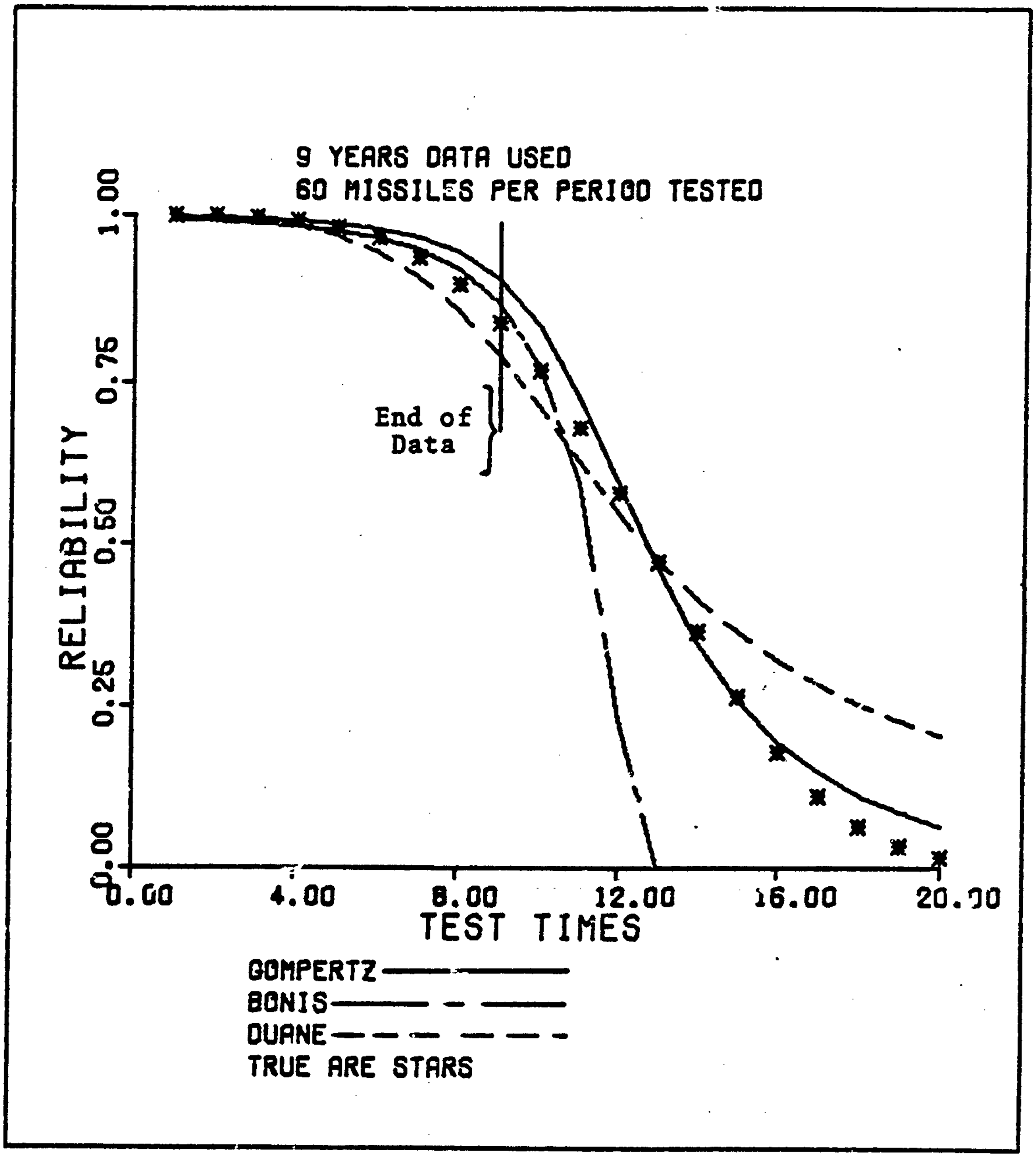


Fig 7. Comparison of Mean Estimated Curves
(9 time periods - 60 missiles)

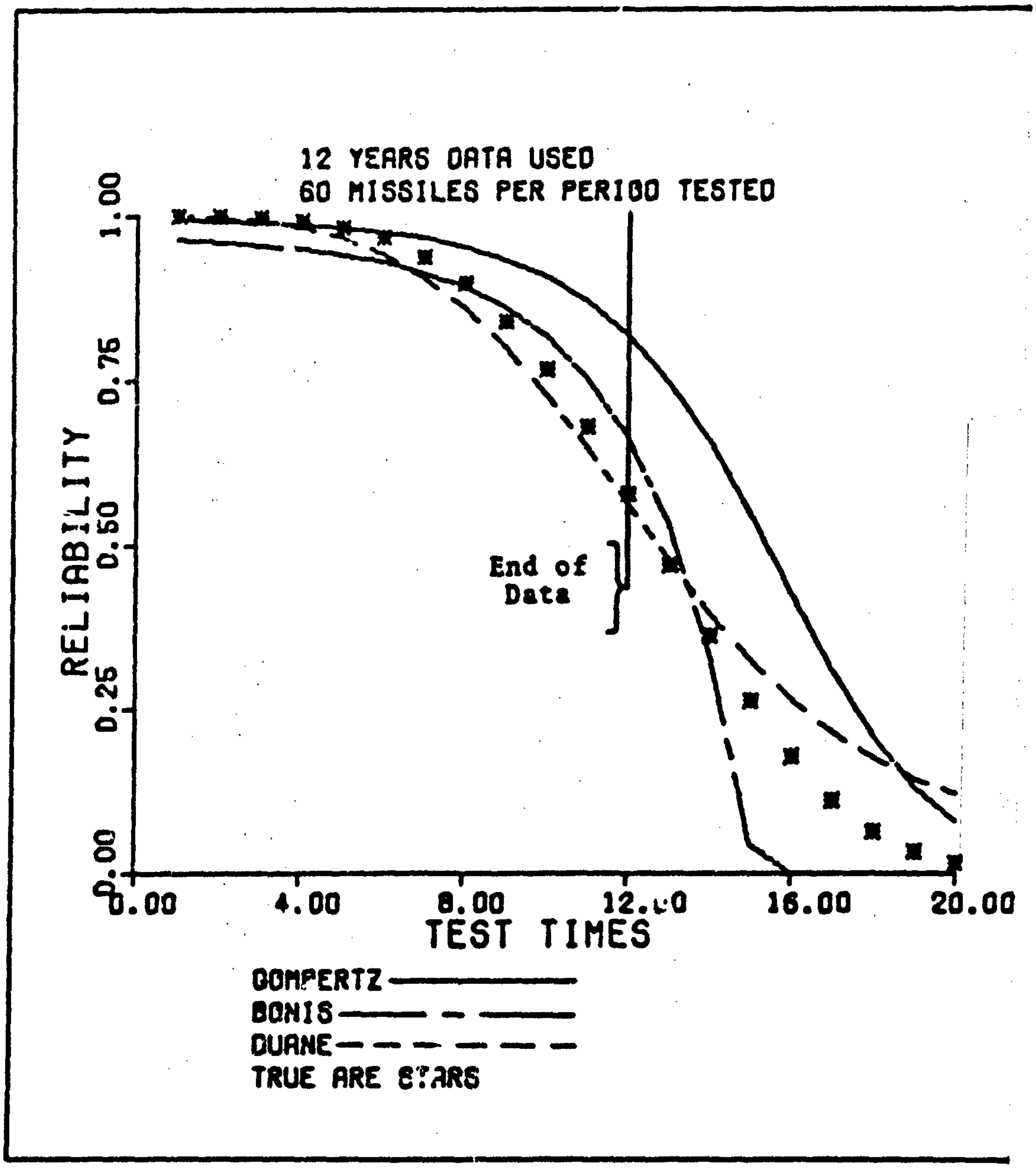


Fig 8. Comparison of Mean Estimated Curves
(12 time periods - 60 missiles)

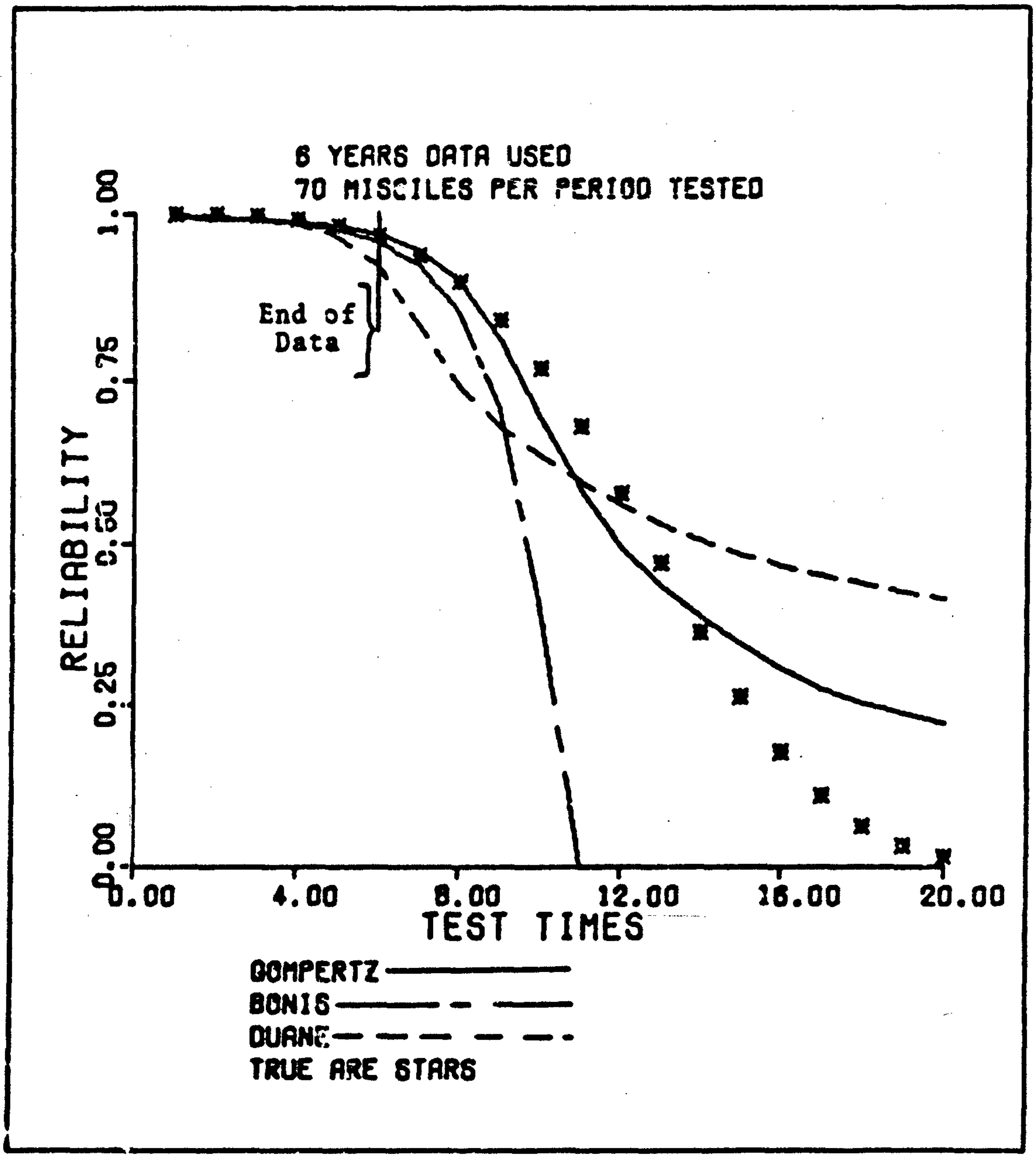


Fig 9. Comparison of Mean Estimated Curves
(6 time periods - 70 missiles)

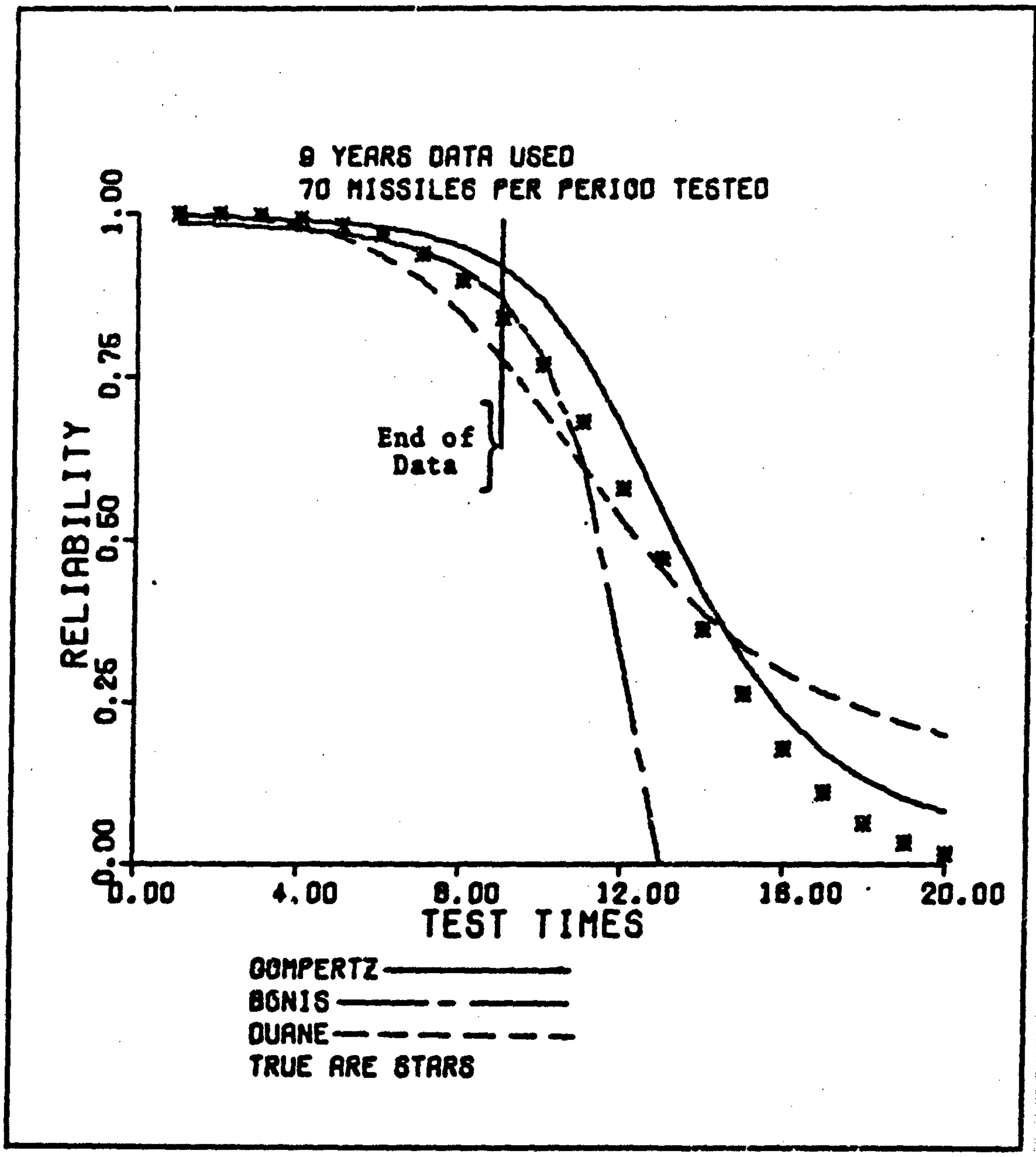


Fig 10. Comparison of Mean Estimated Curves
(9 time periods - 70 missiles)

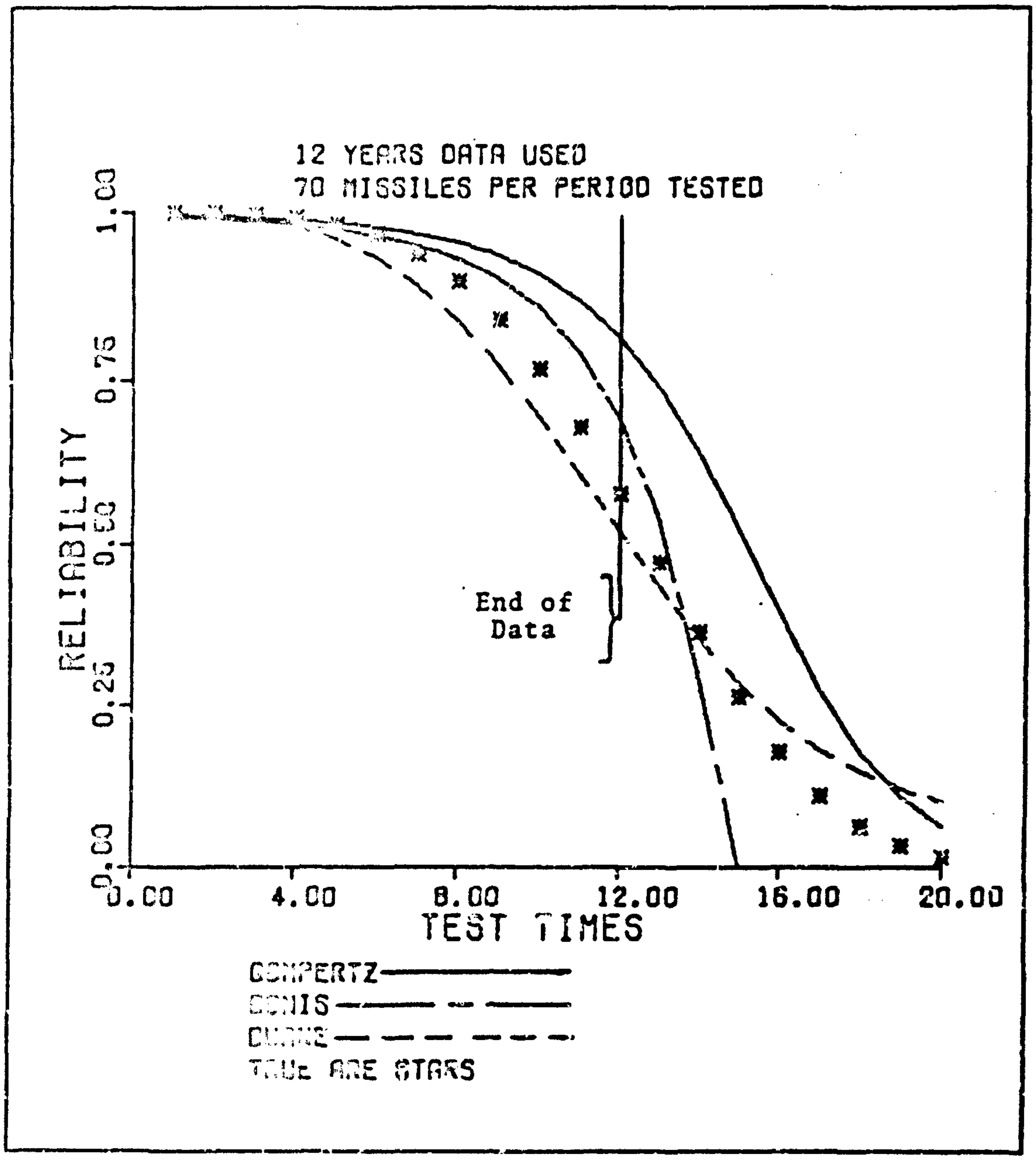


Fig 11. Comparison of Mean Estimated Curves
(12 time periods - 70 missiles)

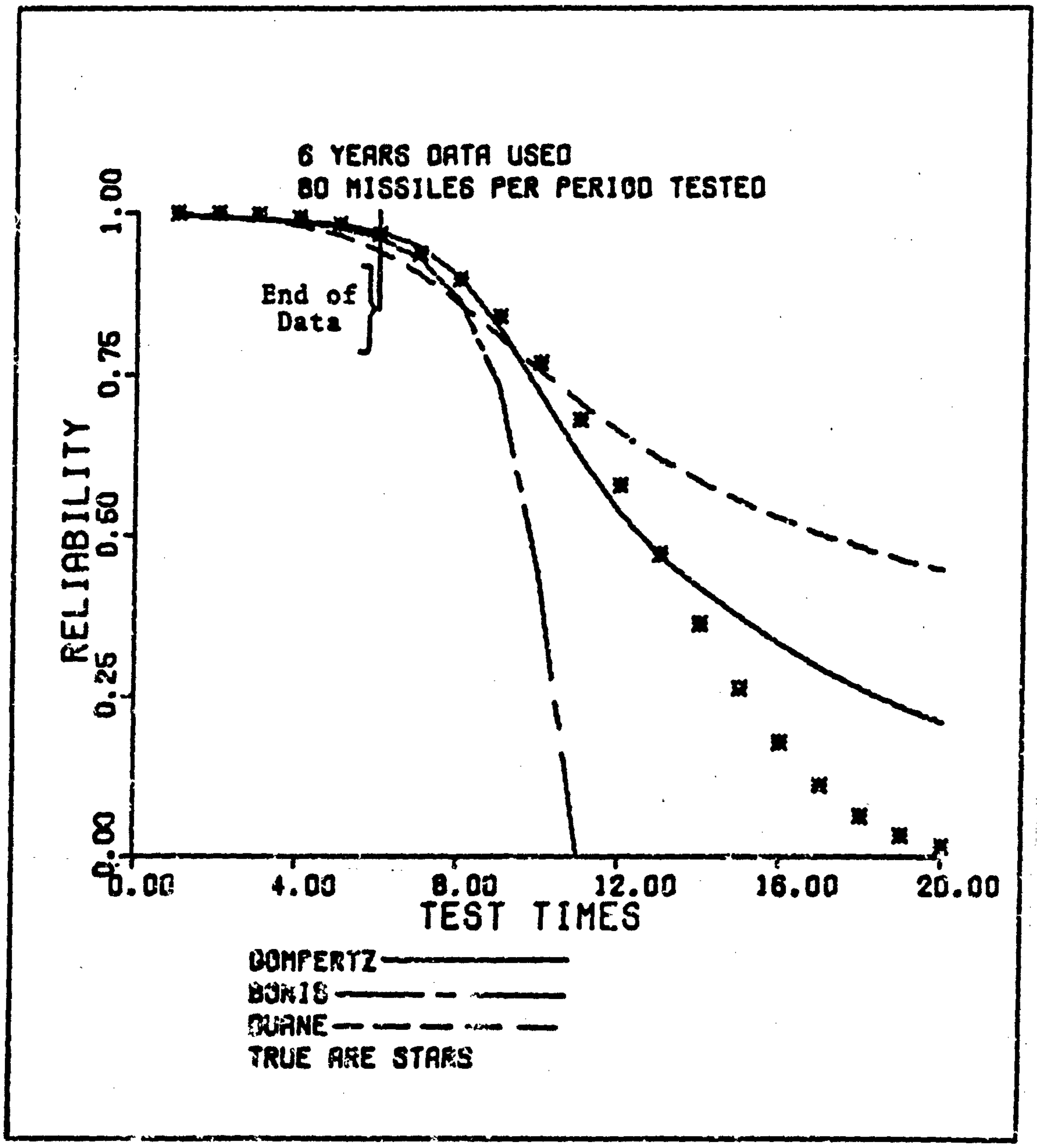


Fig 12. Comparison of Mean Estimated Curves
(6 time periods - 80 missiles)

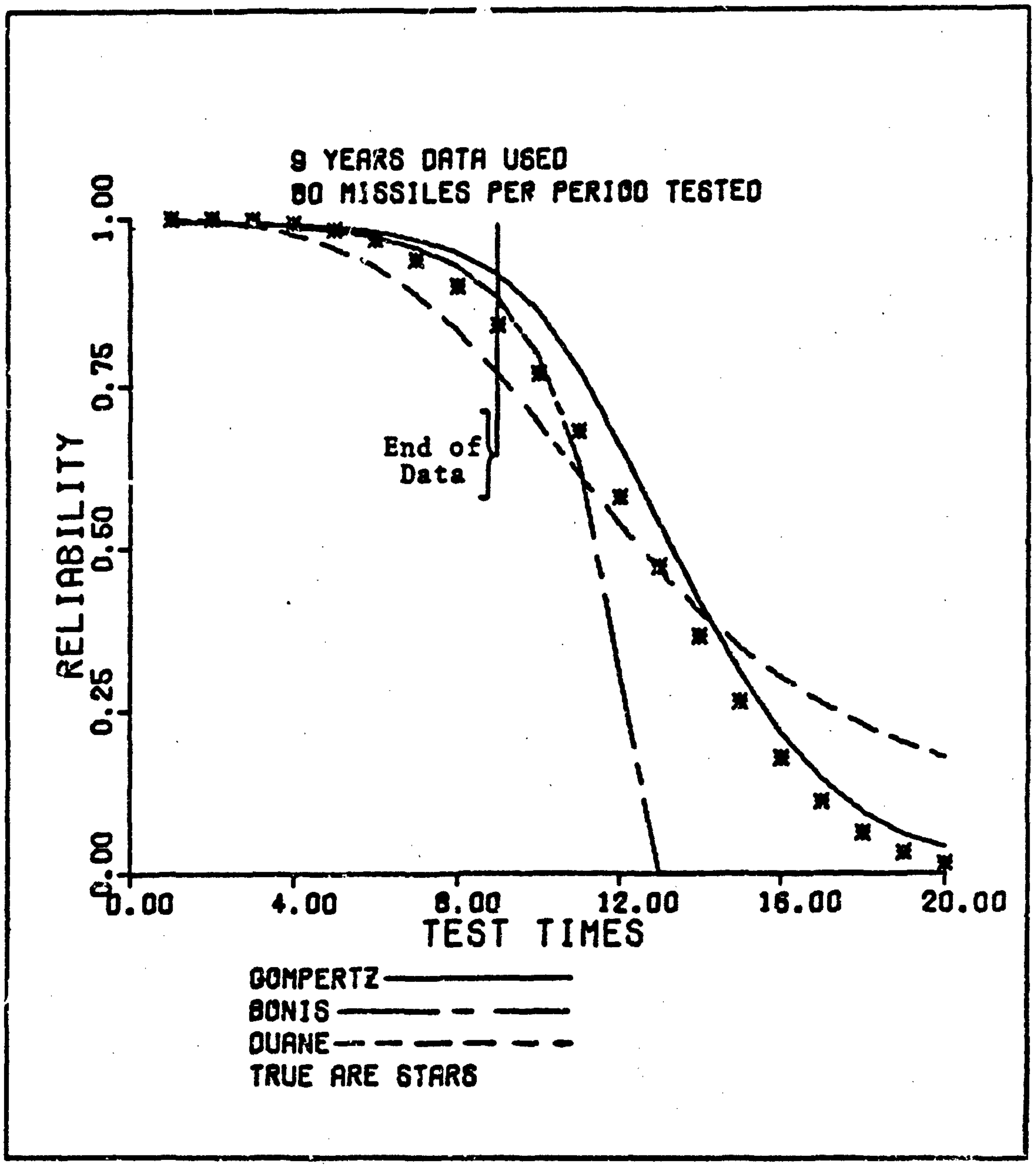


Fig 13. Comparison of Mean Estimated Curves
(9 time periods - 80 missiles)

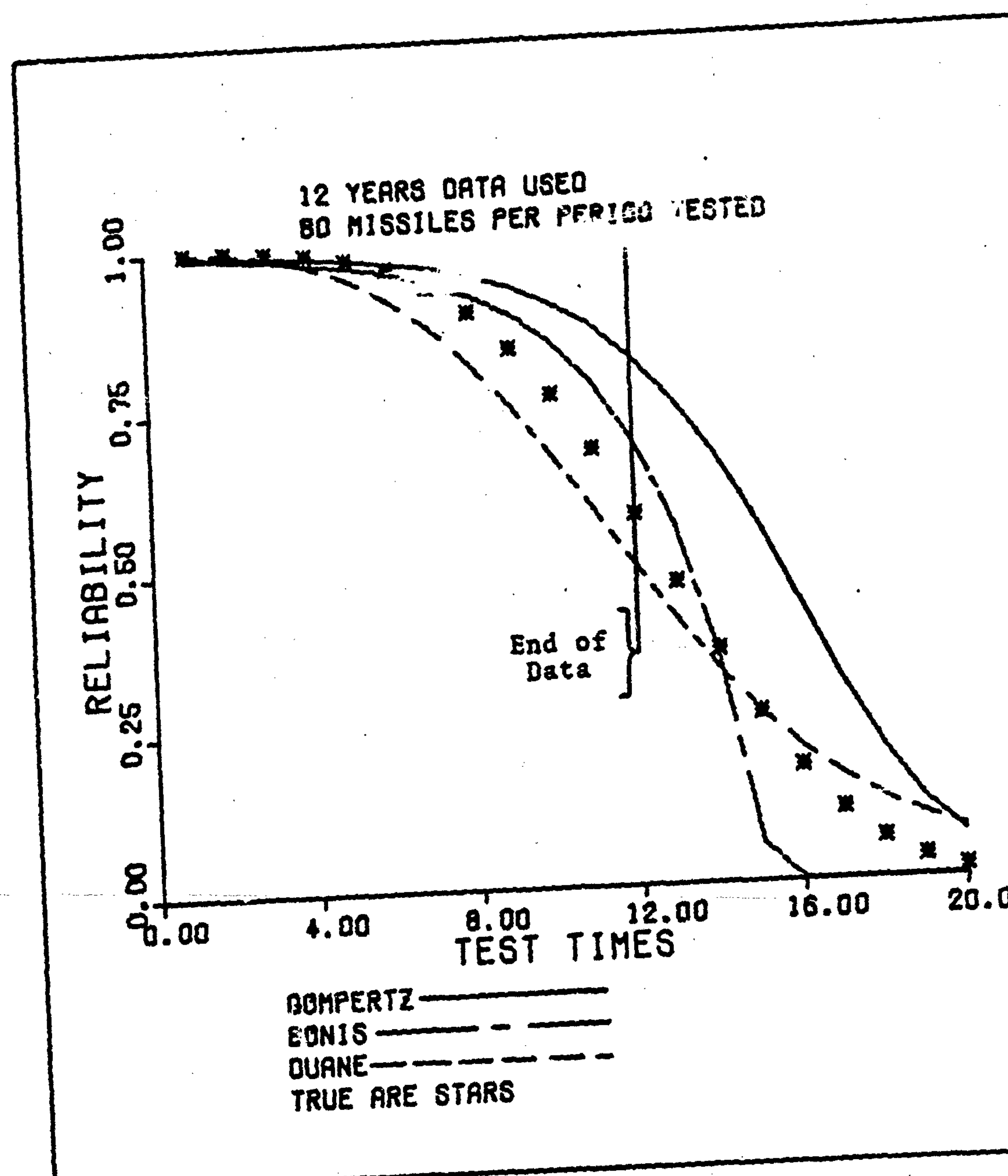


Fig 14. Comparison of Mean Estimated Curves
(12 time periods - 80 missiles)

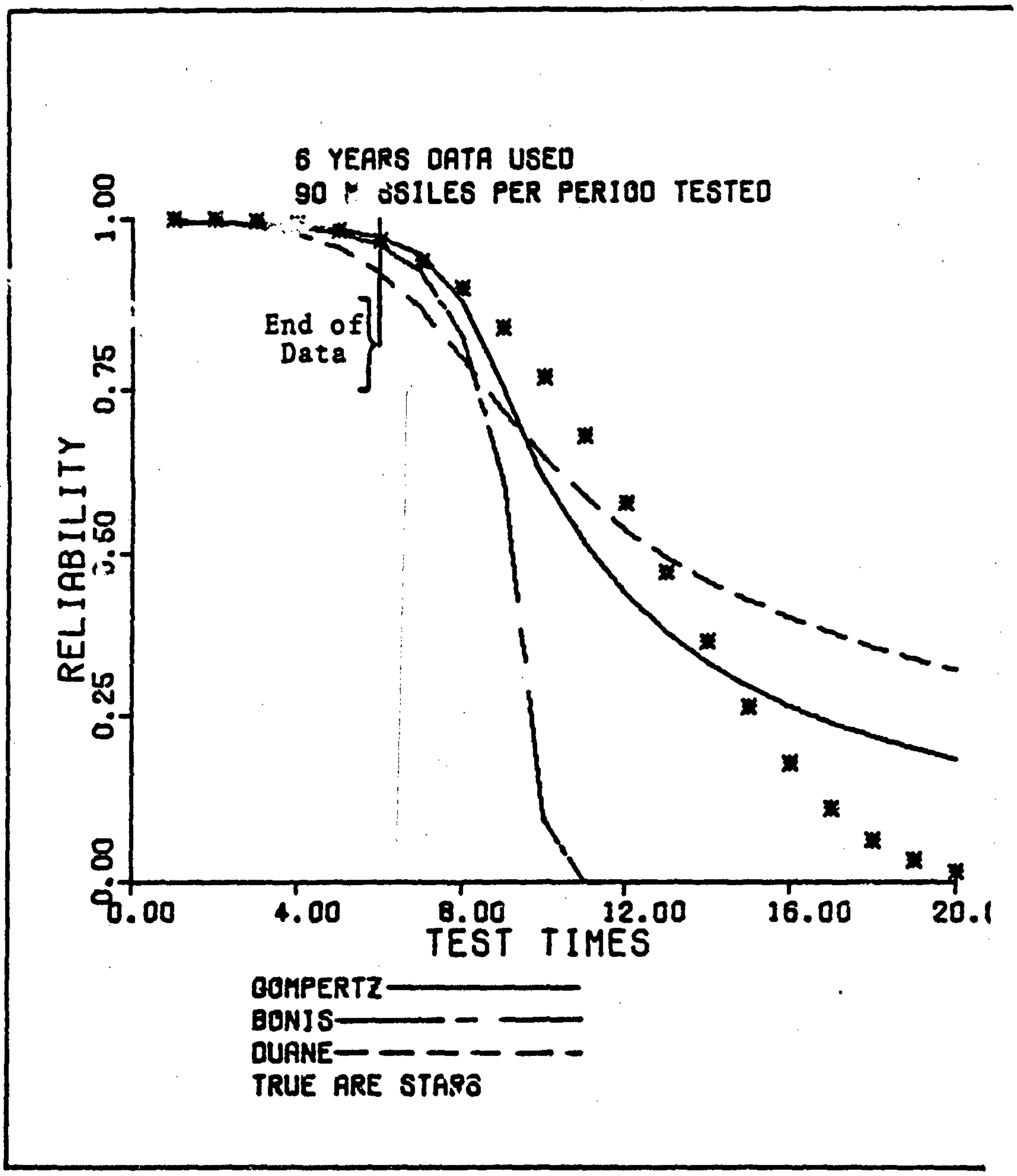


Fig 15. Comparison of Mean Estimated Curves
(6 time periods - 90 missiles)

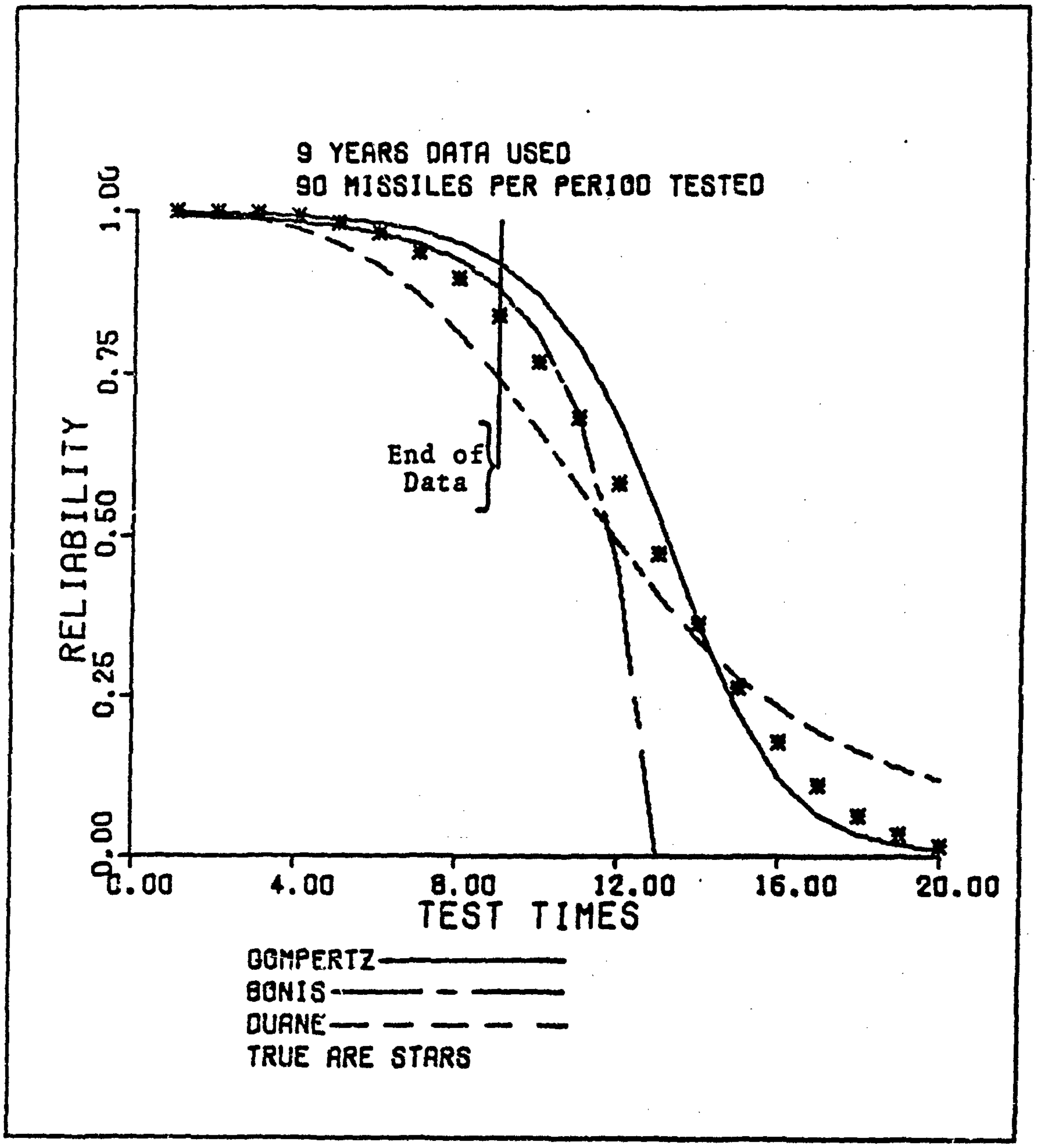


Fig 16. Comparison of Mean Estimated Curves
(9 time periods - 90 missiles)

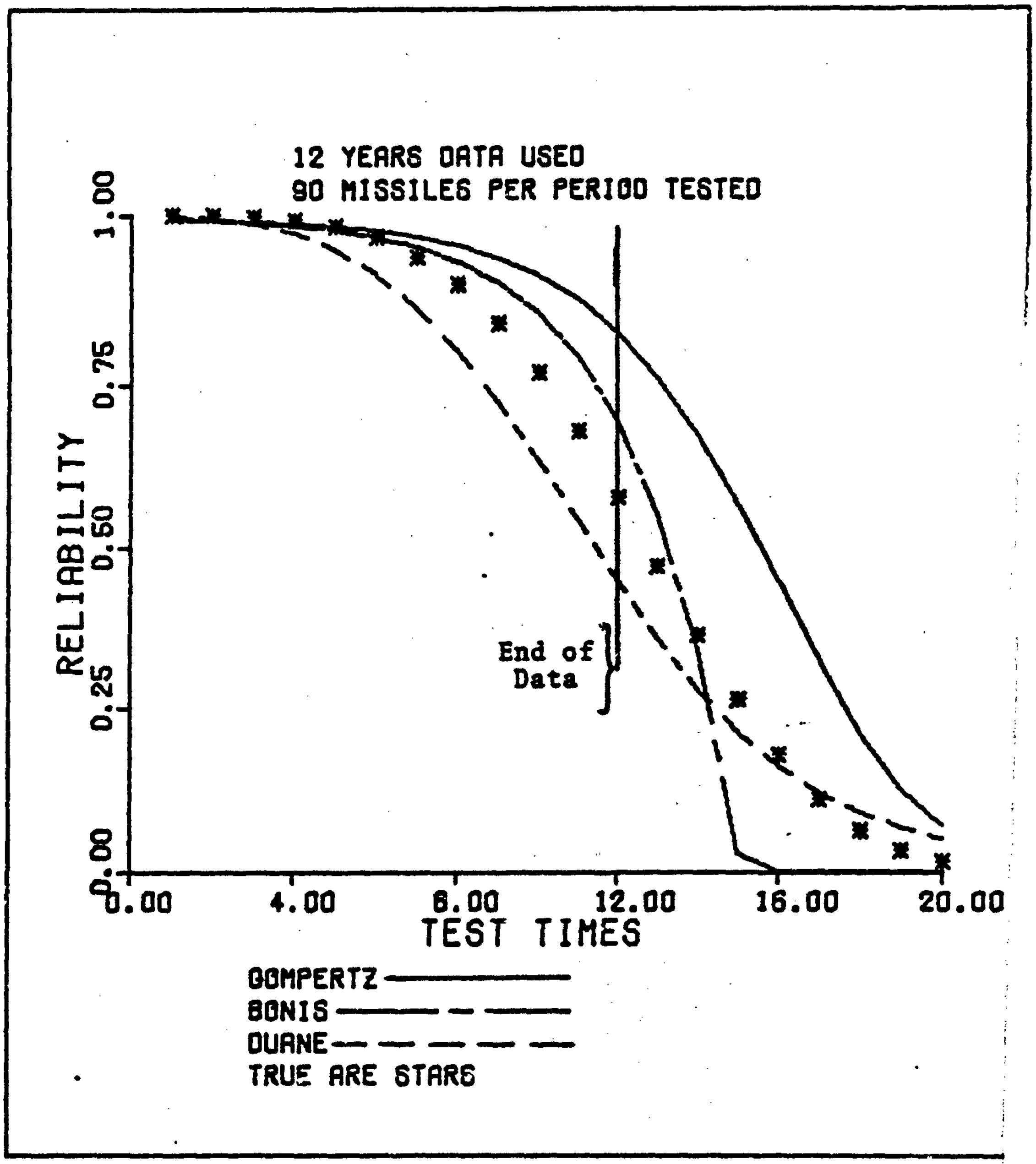


Fig 17. Comparison of Mean Estimated Curves (12 time periods - 90 missiles)

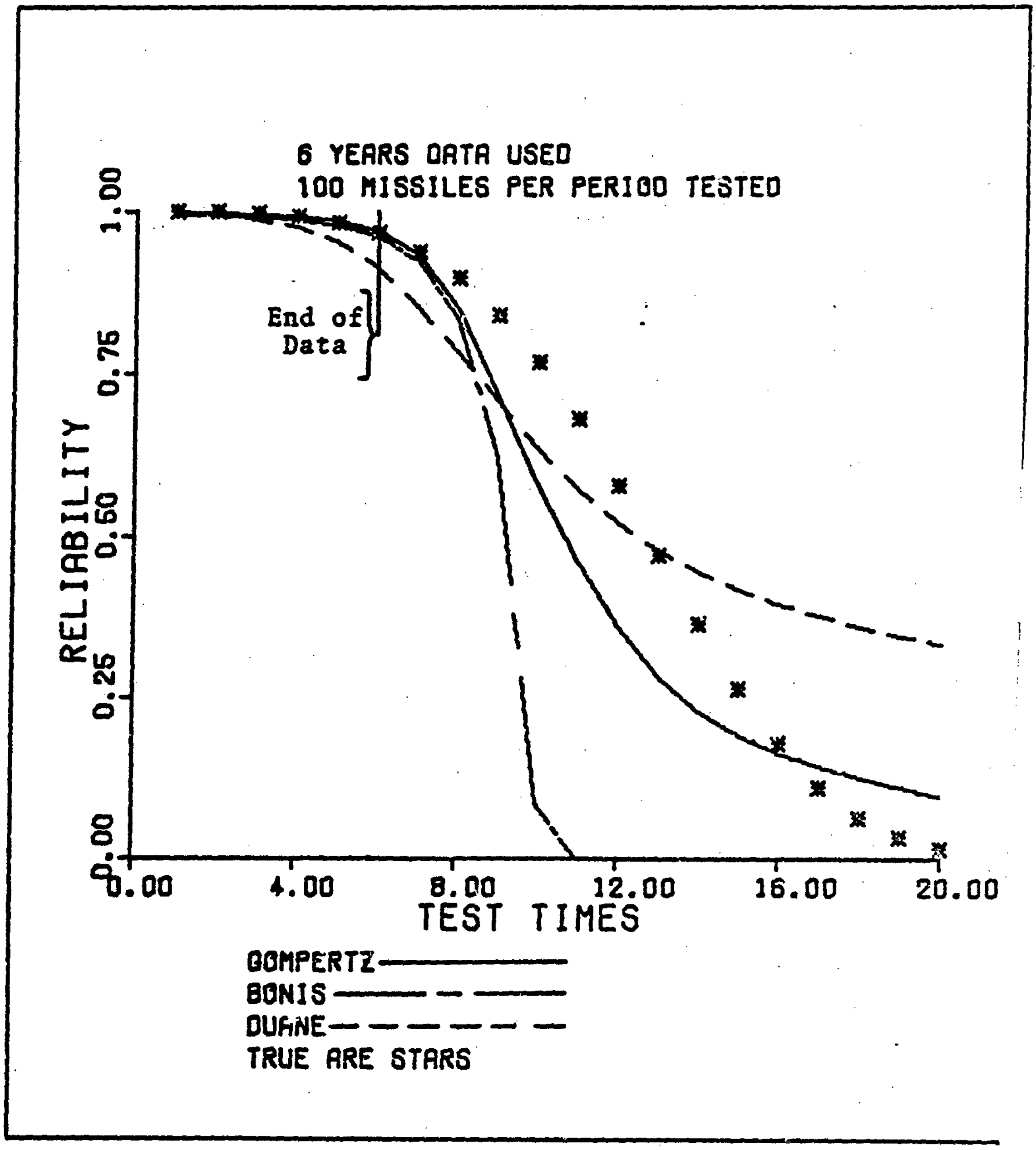


Fig 18. Comparison of Mean Estimated Curves
(6 time periods - 100 missiles)

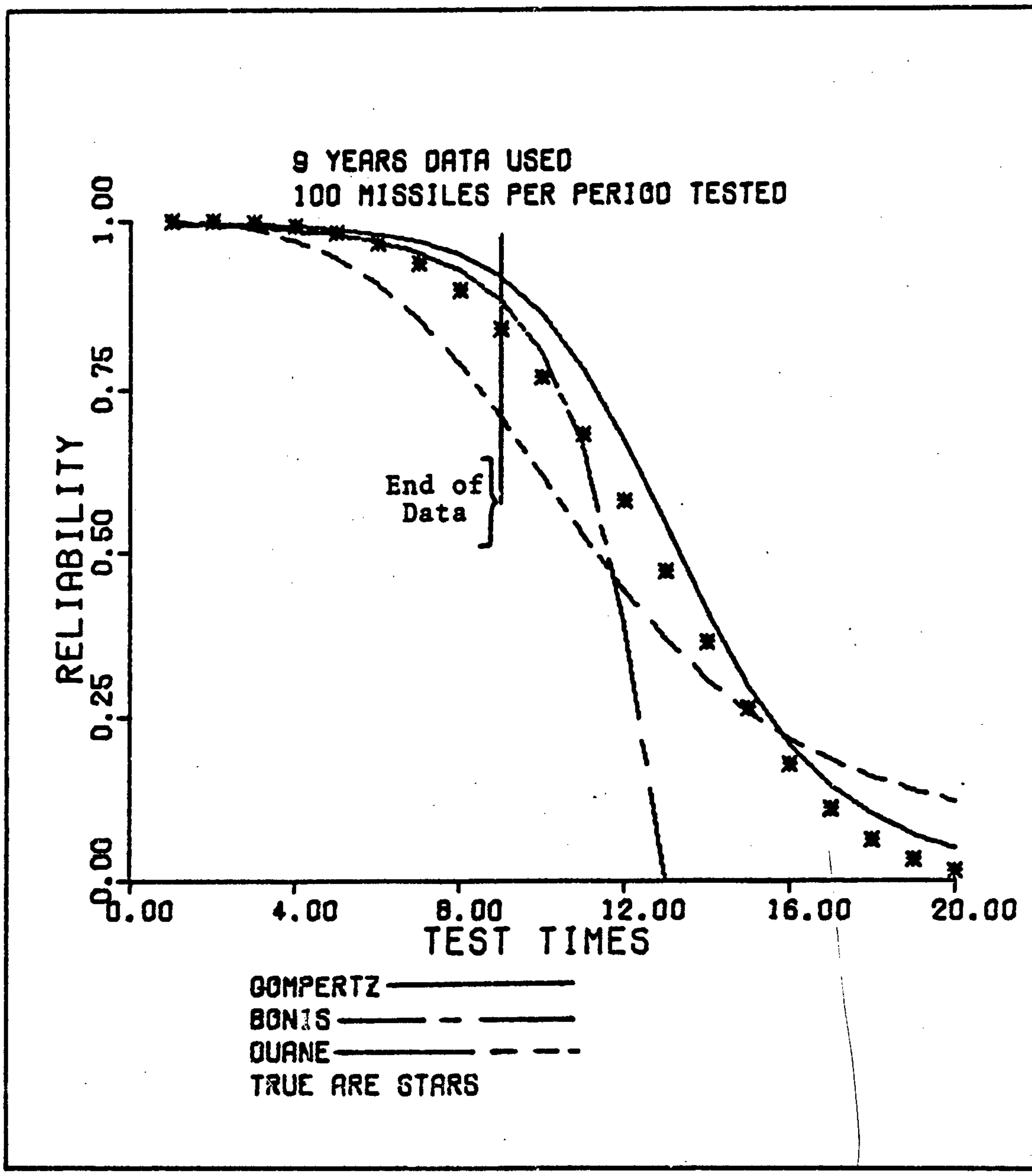


Fig 19. Comparison of Mean Estimated Curves
(9 time periods - 100 missiles)

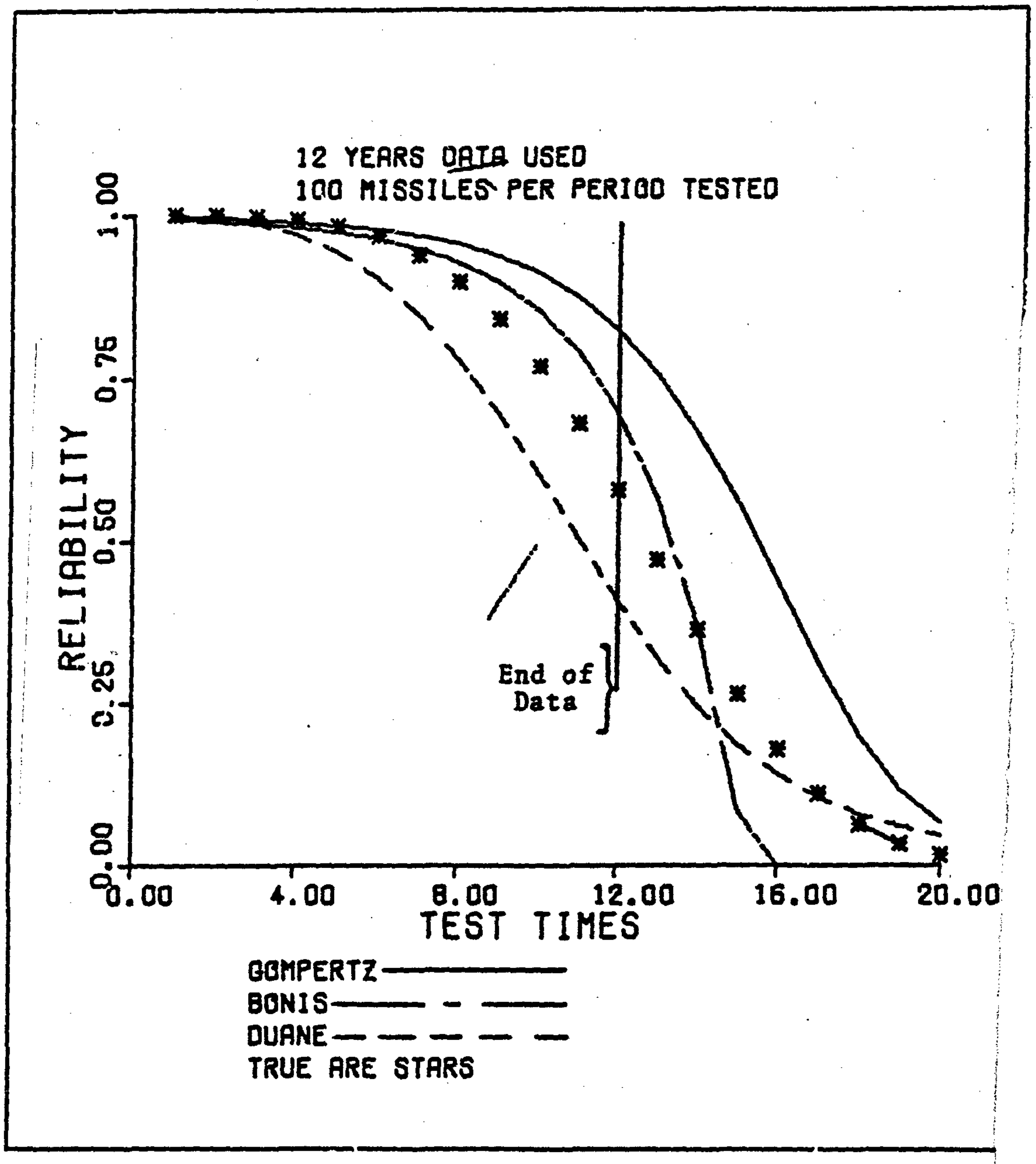


Fig 20. Comparison of Mean Estimated Curves (12 time periods - 100 missiles)

TABLE V

50 Missiles Per Period Tested

Six Periods Data Used			
Model	M1	M2	M3
Gompertz	1.388	.234	.054
Bonis	3.502	1.463	.461
Duane	3.486	1.409	.269
Nine Periods Data Used			
Model	M1	M2	M3
Gompertz	.898	.058	.007
Bonis	1.965	.553	.222
Duane	1.913	.429	.080
Twelve Periods Data Used			
Model	M1	M2	M3
Gompertz	2.506	.534	.083
Bonis	1.262	.164	.069
Duane	1.440	.216	.037

TABLE VI

60 Missiles Per Period Tested

Six Periods Data Used			
Model	M1	M2	M3
Gompertz	1.791	.419	.102
Bonis	2.826	1.031	.334
Duane	3.167	1.078	.216
Nine Periods Data Used			
Model	M1	M2	M3
Gompertz	.528	.024	.004
Bonis	2.035	.602	.221
Duane	1.348	.182	.037
Twelve Periods Data Used			
Model	M1	M2	M3
Gompertz	2.372	.498	.035
Bonis	1.299	.126	.043
Duane	.882	.068	.013

TABLE VII

70 Missiles Per Period Tested

Six Periods Data Used			
Model	M1	M2	M3
Gompertz	1.361	.196	.043
Bonis	3.360	1.425	.461
Duane	2.961	.814	.159
Nine Periods Data Used			
Model	M1	M2	M3
Gompertz	.971	.068	.010
Bonis	1.955	.543	.221
Duane	1.354	.164	.034
Twelve Periods Data Used			
Model	M1	M2	M3
Gompertz	2.188	.429	.076
Bonis	1.260	.165	.069
Duane	.908	.058	.008

TABLE VIII

80 Missiles Per Period Tested

Six Periods Data Used			
Model	M1	M2	M3
Gompertz	1.310	.198	.040
Bonis	3.268	1.388	.461
Duane	2.469	.982	.185
Nine Periods Data Used			
Model	M1	M2	M3
Gompertz	.769	.047	.009
Bonis	1.948	.549	.221
Duane	1.372	.155	.028
Twelve Periods Data Used			
Model	M1	M2	M3
Gompertz	2.425	.532	.093
Bonis	1.143	.132	.042
Duane	.975	.067	.009

TABLE IX

90 Missiles Per Period Tested

Six Periods Data Used			
Model	**M1**	**M2**	**M3**
Gompertz	1.459	.194	.028
Bonis	3.765	1.774	.461
Duane	2.316	.481	.091
Nine Periods Data Used			
Model	**M1**	**M2**	**M3**
Gompertz	.748	.053	.012
Bonis	1.790	.492	.221
Duane	1.226	.104	.012
Twelve Periods Data Used			
Model	**M1**	**M2**	**M3**
Gompertz	2.468	.552	.066
Bonis	1.189	.148	.055
Duane	1.188	.113	.019

TABLE X

100 Missiles Per Period Tested

Six Periods Data Used			
Model	M1	M2	M3
Gompertz	1.481	.226	.048
Bonis	3.769	1.785	.466
Duane	2.402	.502	.100
Nine Periods Data Used			
Model	M1	M2	M3
Gompertz	.801	.052	.010
Bonis	1.852	.510	.221
Duane	1.490	.160	.024
Twelve Periods Data Used			
Model	M1	M2	M3
Gompertz	2.395	.522	.092
Bonis	1.132	.126	.032
Duane	1.473	.183	.031

VII. Conclusions and Recommendations

Military engineers face many important reliability problems. New weapon systems are being designed continually, and many of these systems are to be placed in long-term storage or dormancy. These facts make both reliability growth and dormant reliability important concerns to military analysts.

This study provides evidence that a common attribute of reliability growth and dormant reliability (both are assumed to have changing failure rates) can be the basis of modifying reliability growth methodology for predicting dormant reliability. In Chapter III, the methodology used in reliability growth analysis is presented. Included in this section are the mathematical developments and examples of how to use five well-known reliability growth models. Also, general descriptions of many other reliability growth models are outlined. This section was the foundation for the Monte Carlo experiments discussed in Chapters V and VI. These two chapters tie together the efforts of this study by presenting results of attempts to use reliability growth models to predict dormant reliability.

The results of the simulation indicate that it is possible (at least potentially) to predict dormant reliability with reliability growth models. The graphs shown in

Chapter VI indicate good reliability prediction in many cases. However, it is important to point out that some of the underlying assumptions of the reliability growth models had to be modified before they were used to predict dormant reliability. In addition, inconsistencies were encountered with the Duane and Gompertz models which could not be resolved analytically. These points emphasize the fact that additional work in this area is needed before any specific conclusion can be made.

Recommendations

This study is believed to be the first attempt to use reliability growth models to predict dormant reliability. Of course, this means there are many areas which deserve further study. It is recommended that the reader who wishes to do additional work in this area consider the following:

1) Explore the problem of the shift in the predicted dormant reliability curves that was notices with both the Duane and the Gompertz models.
2) Use Monte Carlo techniques to generate dormant data to be modeled by other reliability growth models. The time series analysis suggested by Singpurwalla (Ref 53) appears to have excellent potential.
3) Analyze the results of using other failure distributions for each of the subsystems of the HM-1.
4) Develop confidence bound for the predicted dormant reliability of the HM-1 using Monte Carlo techniques.

It is also recommended that this study be used as:

1) An introduction to reliability growth. Chapter III can be considered an introductory text to reliability growth and reliability growth modeling.
2) A major reference source for reliability growth.
3) An example for further Monte Carlo analysis of dormant reliability using reliability growth models.

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APPENDIX: SIMULATION DATA

TABLE XI

Gompertz Means and Standard Deviations When 50 Missiles per Period Were Used

	6 Periods Data		9 Periods Data		12 Periods Data	
t	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.991	.012	.996	.005	.994	.006
2	.990	.013	.995	.006	.992	.007
3	.987	.013	.993	.007	.990	.009
4	.983	.012	.990	.008	.986	.010
5	.976	.011	.986	.010	.981	.012
6	.964	.015	.979	.011	.975	.014
7	.941	.035	.967	.011	.966	.015
8	.898	.085	.947	.014	.953	.016
9	.823	.175	.913	.031	.934	.016
10	.732	.279	.852	.078	.908	.015
11	.643	.345	.756	.163	.870	.019
12	.568	.370	.634	.253	.817	.040
13	.500	.379	.518	.291	.744	.078
14	.437	.386	.413	.292	.651	.127
15	.379	.394	.318	.284	.541	.168
16	.332	.403	.240	.268	.434	.188
17	.299	.407	.180	.245	.329	.192
18	.276	.406	.137	.219	.236	.186
19	.260	.405	.105	.191	.163	.170
20	.246	.405	.080	.163	.110	.149

Number of Invalid Estimates Out of 50 Tries

	6 Periods Data	9 Periods Data	12 Periods Data
	34	4	0

TABLE XII

Gompertz Means and Standard Deviations
When 60 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.991	.013	.997	.003	.995	.004
2	.990	.013	.996	.004	.993	.005
3	.988	.014	.993	.003	.991	.005
4	.985	.013	.992	.005	.987	.007
5	.979	.012	.987	.006	.983	.008
6	.967	.016	[illegible]	.007	.976	.009
7	.945	.027	[illegible]	.007	.966	.010
8	.901	.072	[illegible]	.012	.953	.011
9	.825	[illegible]	[illegible]	.033	.934	.012
10	.726	[illegible]	[illegible]	[illegible]	.907	.012
11	.627	[illegible]	[illegible]	[illegible]	.869	.015
12	.542	.376	[illegible]	[illegible]	.817	.027
13	.478	.421	[illegible]	.270	.746	.050
14	.432	[illegible]	[illegible]	.297	.654	.083
15	.400	[illegible]	.237	[illegible]	.544	.121
16	.378	[illegible]	.173	[illegible]	.423	.150
17	.364	[illegible]	.146	.253	.307	.158
18	.353	.444	.111	[illegible]	.207	.149
19	.344	.457	[illegible]	.177	.129	.129
20	.334	[illegible]	.063	.150	.076	.105

Number of Invalid Estimates Out of 50 Tries		
6 Periods Data	9 Periods Data	12 Periods Data
23	0	0

TABLE XIII

Gompertz Means and Standard Deviations
When 70 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.995	.010	.996	.005	.996	.004
2	.994	.010	.995	.006	.994	.005
3	.992	.010	.993	.006	.992	.006
4	.989	.010	.990	.007	.989	.007
5	.983	.010	.986	.008	.984	.008
6	.971	.015	.979	.008	.977	.009
7	.946	.038	.968	.008	.968	.011
8	.897	.094	.949	.011	.955	.012
9	.809	.193	.917	.023	.935	.012
10	.692	.303	.865	.056	.908	.012
11	.582	.371	.781	.119	.868	.014
12	.496	.403	.667	.198	.813	.025
13	.434	.411	.538	.255	.737	.048
14	.388	.403	.417	.281	.638	.082
15	.345	.395	.314	.283	.519	.122
16	.306	.391	.233	.270	.390	.154
17	.275	.388	.173	.252	.271	.164
18	.253	.380	.131	.231	.176	.150
19	.237	.370	.101	.212	.108	.123
20	.221	.360	.080	.195	.063	.095

Number of Invalid Estimates Out of 50 Tries			
	6 Periods Data	9 Periods Data	12 Periods Data
	13	0	0

TABLE XIV

Gompertz Means and Standard Deviations When 80 Missiles per Period Were Used

	6 Periods Data		9 Periods Data		12 Periods Data	
t	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.994	.010	.997	.005	.995	.003
2	.993	.011	.996	.006	.994	.004
3	.991	.013	.994	.007	.991	.005
4	.988	.011	.991	.007	.988	.006
5	.982	.010	.986	.008	.983	.006
6	.971	.013	.979	.008	.976	.007
7	.948	.036	.968	.008	.967	.008
8	.901	.098	.948	.010	.953	.009
9	.820	.202	.915	.027	.934	.009
10	.721	.288	.859	.070	.908	.009
11	.622	.346	.772	.142	.871	.010
12	.534	.382	.658	.215	.821	.018
13	.468	.396	.536	.252	.754	.033
14	.416	.392	.416	.259	.667	.056
15	.372	.383	.306	.247	.560	.080
16	.331	.372	.214	.224	.439	.112
17	.293	.361	.147	.193	.316	.129
18	.261	.349	.095	.164	.206	.127
19	.233	.336	.061	.143	.123	.112
20	.209	.322	.040	.128	.067	.091

Number of Invalid Estimates Out of 50 Trials

6 Periods Data	9 Periods Data	12 Periods Data
4	0	0

TABLE XV

Gompertz Means and Standard Deviations
When 90 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.995	.009	.998	.002	.995	.004
2	.995	.009	.997	.002	.994	.005
3	.993	.007	.995	.003	.991	.006
4	.990	.009	.993	.004	.988	.007
5	.985	.009	.988	.005	.983	.008
6	.973	.012	.981	.006	.976	.009
7	.944	.037	.970	.008	.967	.011
8	.878	.110	.951	.011	.954	.012
9	.731	.231	.920	.018	.936	.013
10	.613	.364	.869	.035	.910	.013
11	.515	.395	.790	.066	.874	.014
12	.438	.405	.674	.113	.825	.019
13	.378	.404	.525	.164	.759	.031
14	.330	.398	.364	.189	.672	.051
15	.293	.391	.224	.179	.566	.078
16	.263	.380	.123	.150	.444	.106
17	.239	.370	.063	.114	.319	.126
18	.217	.359	.032	.081	.208	.129
19	.199	.347	.016	.053	.124	.114
20	.182	.335	.008	.032	.069	.090

Number of Invalid Estimates Out of 50 Tries

	6 Periods Data	9 Periods Data	12 Periods Data
	19	0	0

TABLE XVI

Gompertz Means and Standard Deviations When 100 Missiles per Period Were Used

	6 Periods Data		9 Periods Data		12 Periods Data	
t	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.998	.003	.997	.003	.995	.003
2	.997	.003	.996	.003	.994	.004
3	.996	.003	.994	.004	.991	.005
4	.992	.004	.992	.005	.988	.006
5	.986	.005	.987	.006	.983	.007
6	.970	.017	.980	.007	.974	.008
7	.932	.059	.969	.007	.967	.009
8	.849	.156	.950	.010	.954	.010
9	.716	.276	.917	.039	.935	.010
10	.577	.347	.862	.081	.909	.009
11	.460	.365	.780	.140	.872	.009
12	.340	.364	.670	.186	.822	.015
13	.283	.355	.539	.234	.754	.028
14	.228	.342	.407	.260	.666	.050
15	.190	.326	.294	.260	.551	.078
16	.163	.308	.208	.240	.433	.107
17	.142	.290	.146	.210	.308	.127
18	.124	.272	.102	.179	.197	.131
19	.109	.254	.071	.148	.116	.118
20	.095	.238	.049	.119	.065	.096

Number of Invalid Estimates Out of 50 Tries

6 Periods Data	9 Periods Data	12 Periods Data
9	0	0

TABLE XVII

Bonis Means and Standard Deviations When 50 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.922	.011	.991	.017	.911	.013
2	.989	.015	.988	.019	.988	.015
3	.985	.015	.985	.022	.984	.018
4	.978	.017	.980	.025	.978	.021
5	.966	.017	.974	.029	.971	.025
6	.945	.018	.963	.032	.961	.030
7	.906	.036	.947	.035	.947	.035
8	.830	.100	.920	.035	.926	.040
9	.678	.262	.872	.036	.897	.045
10	.363	.641	.786	.067	.854	.047
11			.623	.193	.790	.044
12			.300	.542	.693	.040
13					.542	.080
14					.303	.220
15						
16						
17						
18						
19						
20						

Means are negative in this area of the Table, therefore, calculations are not considered

Number of Invalid Estimates Out of 50 Tries		
6 Periods Data	9 Periods Data	12 Periods Data
43	7	1

TABLE XVIII

Bonis Means and Standard Deviations
When 60 Missiles per Period Were Used

t	6 Periods Data Mean	6 Periods Data Std-Dv	9 Periods Data Mean	9 Periods Data Std-Dv	12 Periods Data Mean	12 Periods Data Std-Dv
1	.995	.007	.994	.011	.964	.173
2	.923	.008	.991	.013	.961	.177
3	.989	.010	.988	.015	.956	.180
4	.984	.011	.984	.018	.950	.183
5	.974	.012	.977	.021	.941	.186
6	.957	.013	.966	.024	.930	.188
7	.923	.021	.949	.026	.914	.190
8	.875	.057	.920	.026	.892	.192
9	.777	.142	.878	.030	.860	.192
10	.594	.337	.774	.076	.816	.189
11	.242	.776	.592	.223	.753	.184
12			.226	.601	.662	.174
13					.529	.164
14					.334	.177
15					.043	.276
16						
17	Means are negative in this area of the Table, therefore, calculations are not considered					
18						
19						
20						

Number of Invalid Estimates Out of 50 Tries		
6 Periods Data	9 Periods Data	12 Periods Data
31	1	0

TABLE XIX

Bonis Means and Standard Deviations
When 70 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.996	.006	.985	.064	.993	.007
2	.994	.008	.983	.066	.990	.009
3	.991	.009	.980	.068	.987	.012
4	.986	.010	.976	.070	.982	.014
5	.976	.011	.969	.073	.975	.018
6	.957	.013	.959	.074	.965	.022
7	.922	.027	.942	.075	.951	.026
8	.851	.077	.915	.074	.930	.031
9	.704	.207	.867	.070	.901	.035
10	.396	.531	.781	.073	.856	.037
11			.622	.145	.790	.033
12			.319	.395	.689	.027
13					.533	.067
14					.288	.189
15						
16						
17						
18						
19						
20						

Means are negative in this area of the Table, therefore, calculations are not considered

Number of Invalid Estimates Out of 50 Tries

6 Periods Data	9 Periods Data	12 Periods Data
25	0	0

TABLE XX

Bonis Means and Standard Deviations
When 80 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.996	.006	.996	.005	.991	.012
2	.994	.007	.994	.006	.988	.014
3	.991	.008	.992	.007	.984	.016
4	.986	.009	.988	.009	.979	.019
5	.977	.009	.981	.011	.971	.022
6	.961	.010	.971	.014	.960	.025
7	.923	.025	.955	.016	.945	.028
8	.864	.077	.927	.017	.923	.031
9	.730	.220	.880	.020	.892	.032
10	.438	.600	.795	.058	.848	.032
11			.633	.199	.784	.029
12			.306	.646	.691	.024
13					.555	.045
14					.354	.110
15					.058	.237
16						
17	Means are negative in this area of the Table, therefore, calculations are not considered					
18						
19						
20						

Number of Invalid Estimates Out of 50 Tries

6 Periods Data	9 Periods Data	12 Periods Data
14	0	0

TABLE XXI

Bonis Means and Standard Deviations
When 90 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.997	.005	.993	.010	.994	.003
2	.995	.006	.991	.012	.992	.004
3	.992	.008	.988	.014	.988	.006
4	.988	.009	.983	.016	.984	.007
5	.978	.009	.977	.019	.976	.009
6	.960	.010	.966	.021	.966	.012
7	.920	.029	.950	.023	.952	.015
8	.829	.102	.925	.023	.930	.019
9	.615	.317	.882	.022	.900	.020
10	.096	.929	.810	.040	.855	.022
11			.683	.123	.789	.022
12			.448	.349	.693	.021
13					.551	.032
14					.340	.080
15					.028	.182
16						
17						
18						
19						
20						

Means are negative in this area
of the Table, therefore,
calculations are not considered

Number of Invalid Estimates Out of 50 Tries		
6 Periods Data	9 Periods Data	12 Periods Data
8	0	0

TABLE XXII

Bonis Means and Standard Deviations When 100 Missiles per Period Were Used

t	6 Periods Data Mean	6 Periods Data Std-Dv	9 Periods Data Mean	9 Periods Data Std-Dv	12 Periods Data Mean	12 Periods Data Std-Dv
1	.996	.009	.995	.005	.993	.007
2	.994	.010	.993	.007	.990	.008
3	.991	.012	.990	.009	.986	.010
4	.987	.013	.986	.011	.981	.012
5	.978	.014	.980	.013	.973	.015
6	.960	.014	.969	.016	.963	.017
7	.922	.031	.953	.018	.947	.020
8	.833	.109	.927	.018	.926	.023
9	.619	.349	.882	.019	.895	.024
10	.085	1.072	.804	.047	.851	.025
11			.663	.163	.798	.022
12			.388	.530	.696	.020
13					.563	.035
14					.369	.081
15					.084	.170
16						
17	Means are negative in this area of the Table, therefore, calculations are not considered					
18						
19						
20						

Number of Invalid Estimates Out of 50 Tries

6 Periods Data	9 Periods Data	12 Periods Data
4	0	0

TABLE XXIII

Duane Means and Standard Deviations
When 50 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.998	.002	.999	.001	.999	.001
2	.996	.003	.997	.003	.997	.002
3	.993	.005	.993	.005	.993	.005
4	.987	.008	.985	.007	.984	.008
5	.975	.016	.973	.010	.971	.010
6	.953	.044	.954	.015	.950	.013
7	.911	.122	.926	.026	.922	.016
8	.861	.201	.883	.046	.884	.022
9	.813	.249	.837	.080	.835	.035
10	.767	.286	.774	.125	.776	.057
11	.724	.322	.703	.178	.707	.089
12	.685	.355	.631	.228	.732	.124
13	.653	.379	.563	.267	.555	.157
14	.628	.393	.504	.291	.481	.182
15	.608	.400	.454	.304	.413	.197
16	.591	.405	.411	.310	.353	.205
17	.575	.408	.374	.312	.300	.208
18	.560	.410	.343	.311	.255	.206
19	.546	.413	.313	.301	.217	.201
20	.534	.416	.291	.306	.185	.194

Number of Invalid Estimates Out of 50 Tries

	6 Periods Data	9 Periods Data	12 Periods Data
	9	0	0

TABLE XXIV

Duane Means and Standard Deviations
When 60 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.998	.002	.999	.001	.999	.001
2	.997	.003	.997	.003	.997	.002
3	.993	.004	.993	.004	.992	.004
4	.987	.007	.984	.007	.983	.007
5	.973	.016	.969	.009	.966	.007
6	.942	.054	.945	.014	.941	.012
7	.881	.156	.909	.028	.906	.016
8	.810	.253	.858	.055	.859	.025
9	.749	.308	.791	.097	.799	.041
10	.697	.350	.712	.151	.728	.064
11	.656	.373	.627	.204	.648	.093
12	.623	.389	.545	.246	.562	.124
13	.594	.402	.473	.271	.477	.152
14	.570	.412	.413	.283	.396	.174
15	.549	.419	.361	.287	.325	.188
16	.531	.424	.318	.288	.264	.194
17	.516	.427	.281	.286	.214	.194
18	.502	.430	.251	.282	.175	.191
19	.490	.432	.224	.278	.145	.185
20	.479	.434	.202	.277	.119	.179

Number of Invalid Estimates Out of 50 Tries

6 Periods Data	9 Periods Data	12 Periods Data
2	0	0

TABLE XXV

Duane Means and Standard Deviations
When 70 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.998	.004	.999	.002	.999	.001
2	.996	.005	.996	.004	.977	.003
3	.992	.006	.991	.006	.991	.006
4	.984	.007	.980	.008	.979	.009
5	.966	.016	.963	.010	.960	.012
6	.921	.068	.936	.016	.931	.014
7	.833	.199	.897	.030	.891	.017
8	.740	.316	.844	.056	.838	.025
9	.681	.354	.776	.093	.772	.042
10	.635	.371	.697	.140	.694	.065
11	.595	.384	.611	.189	.608	.093
12	.560	.396	.527	.232	.519	.123
13	.529	.405	.451	.264	.431	.149
14	.504	.412	.387	.284	.351	.168
15	.483	.415	.336	.294	.282	.178
16	.466	.417	.295	.297	.226	.180
17	.450	.418	.263	.297	.181	.175
18	.437	.418	.237	.293	.147	.166
19	.424	.418	.216	.289	.120	.155
20	.413	.417	.198	.284	.099	.143

Number of Invalid Estimates Out of 50 Tries		
6 Periods Data	9 Periods Data	12 Periods Data
0	0	0

TABLE XXVI

Duane Means and Standard Deviations
When 80 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.998	.002	.999	.001	.999	.001
2	.995	.004	.996	.003	.996	.003
3	.989	.005	.989	.006	.989	.005
4	.980	.008	.967	.008	.975	.009
5	.965	.015	.956	.010	.953	.011
6	.941	.035	.926	.015	.920	.013
7	.906	.077	.885	.028	.875	.015
8	.859	.139	.832	.052	.818	.022
9	.807	.194	.766	.086	.750	.036
10	.756	.237	.692	.127	.671	.056
11	.706	.274	.613	.167	.585	.079
12	.660	.305	.536	.201	.497	.102
13	.619	.328	.464	.224	.412	.121
14	.583	.346	.400	.238	.333	.134
15	.551	.358	.346	.243	.265	.140
16	.524	.367	.300	.242	.207	.138
17	.501	.373	.261	.238	.161	.132
18	.480	.376	.228	.232	.125	.123
19	.461	.379	.200	.225	.097	.113
20	.444	.380	.176	.217	.076	.102

Number of Invalid Estimates Out of 50 Tries			
	6 Periods Data	9 Periods Data	12 Periods Data
	0	0	0

TABLE XXVII

Duane Means and Standard Deviations
When 90 Missiles per Period Were Used

	6 Periods Data		9 Periods Data		12 Periods Data	
t	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.998	.002	.999	.001	.999	.001
2	.995	.004	.996	.004	.996	.003
3	.989	.006	.988	.007	.988	.007
4	.977	.008	.974	.010	.973	.011
5	.956	.017	.951	.013	.948	.015
6	.921	.045	.918	.016	.912	.018
7	.867	.101	.872	.024	.863	.019
8	.795	.178	.813	.041	.799	.021
9	.718	.247	.741	.067	.723	.030
10	.648	.292	.660	.100	.635	.047
11	.587	.321	.572	.134	.542	.069
12	.536	.339	.486	.166	.447	.092
13	.492	.350	.406	.190	.358	.110
14	.456	.354	.337	.202	.278	.120
15	.425	.355	.279	.204	.212	.122
16	.398	.354	.231	.201	.160	.117
17	.375	.352	.193	.193	.120	.107
18	.353	.350	.142	.183	.089	.095
19	.334	.348	.138	.172	.067	.082
20	.317	.346	.117	.160	.050	.070

Number of Invalid Estimates Out of 50 Tries			
	6 Periods Data	9 Periods Data	12 Periods Data
	0	0	0

TABLE XXVIII

Duane Means and Standard Deviations
When 100 Missiles per Period Were Used

t	6 Periods Data		9 Periods Data		12 Periods Data	
	Mean	Std-Dv	Mean	Std-Dv	Mean	Std-Dv
1	.998	.004	.999	.001	.999	.001
2	.994	.005	.995	.004	.996	.003
3	.987	.007	.987	.007	.987	.007
4	.975	.009	.971	.011	.970	.011
5	.952	.018	.946	.013	.942	.015
6	.913	.051	.908	.017	.902	.017
7	.852	.128	.856	.031	.848	.018
8	.782	.191	.788	.059	.778	.025
9	.707	.244	.706	.098	.695	.041
10	.635	.293	.616	.142	.601	.064
11	.572	.327	.526	.179	.503	.089
12	.519	.351	.411	.206	.408	.111
13	.477	.365	.367	.221	.321	.125
14	.443	.373	.306	.227	.246	.131
15	.416	.376	.256	.226	.186	.129
16	.393	.376	.216	.220	.140	.121
17	.375	.375	.184	.210	.105	.109
18	.358	.372	.159	.199	.079	.097
19	.343	.369	.138	.188	.060	.085
20	.330	.366	.121	.176	.045	.074

Number of Invalid Estimates Out of 50 Tries		
6 Periods Data	9 Periods Data	12 Periods Data
0	0	0

Vita

John Frederick VonLoh was born on 27 June 1949 in Holdrege, Nebraska, the son of Harold F. H. and Vivian E. VonLoh. He graduated from Bertrand High School in Bertrand, Nebraska. His undergraduate studies were completed at Kearney State College in Kearney, Nebraska, where he received a Bachelor of Science degree in Mathematics. After being commissioned on 24 April 1974 into the United States Air Force, he became a navigator and flew B-52G model aircraft at Seymour-Johnson Air Force Base for five years.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This thesis presents the results of an extensive literature search into reliability growth and the subsequent use of three reliability growth models to predict dormant reliability. A brief review of reliability theory is followed by a survey of reliability growth models, which

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includes the detailed developments of and specific examples for five popular models. The nature of dormant reliability is then discussed as a prelude to a Monte Carlo analysis using the Duane, Gompertz, and Bonis reliability growth models to predict dormant reliability.